AN AUTOMATIC IMPEDANCE MATCHING SYSTEM FOR TRANSMISSION LINES

JOSEPH E. FENWICK

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AN AUTOMATIC IMPEDANCE MATCHING SYSTEM
FOR
TRANSMISSION LINES

by

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Submitted in partial fulfillment
of the requirements
for the degree of
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PREFACE

Perhaps the most difficult problem in the design of microwave transmission systems is that of maintaining a low voltage standing wave ratio over wide frequency bandwidths. The solution of this problem becomes more and more critical as the power levels at which such systems operate are increased, due largely to the adverse effects of the reflected power on the microwave generator.

The availability of a reliable and practical automatic impedance matching system would reduce the difficulty of solving this design problem significantly by increasing the maximum allowable voltage standing wave ratio of each component of the transmission system.

This paper presents qualitative and mathematical analyses of an automatic impedance matching system which is fundamentally unrestricted in power level and is capable of matching values of load voltage standing wave ratio much larger than are usually encountered in microwave transmission systems. The system proposed and analysed is inherently adaptable to any type of microwave or ultra-high-frequency transmission system and is specifically not restricted to use in waveguide systems, although the system which has been constructed and tested is a waveguide system.

In addition, this paper presents experimental evidence in support of the analyses and, briefly, the development of

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an automatic impedance matching system for use in a waveguide transmission system operating at X-band frequencies.

The experimental work preceeding the preparation of this paper and the development of the X-band impedance matching system, which is known as the Susceptance Probe Tuner, were done in the Research Laboratory of the Dalmo Victor Company, San Carlos, California, during the author's industrial experience tour of 3 January 1955 to 18 March 1955. The author is particularly indebted to Mr. Glenn A. Walters, Director of the Dalmo Victor Research Laboratory, and to Mr. Robert R. Johnson, head of the Research Laboratory Circuit Design Section, for their interest and assistance in the development of the Susceptance Probe Tuner. The assistance and cooperation of the personnel of the Dalmo Victor Research Laboratory are gratefully acknowledged.

Joseph E. Fenwick

MONTEREY, CALIFORNIA

June, 1955

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LIST OF ILLUSTRATIONS

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TABLE OF SYMBOLS AND ABBREVIATIONS

(Listed in the order of their appearance in the text)

VSWR	Voltage standing wave ratio
β	The imaginary part of the complex propagation constant, radians per unit physical length
ℓ	Physical length
$\beta\ell$	Electrical length
M	The position of match on the transmission line
b_{s_M}	The match value of the stub susceptance, normalized units
y_L	The admittance of the load on the transmission line, normalized units
λ	Wavelength along the transmission line
ω_m	The angular modulation frequency, radians per second
t	The time variable
γ	The complex propagation constant
$\overline{\beta\ell}$	The average electrical distance from the load to the stub, positive when measured in the direction from the load to the generator; the time-average value of $\beta\ell$
$\Delta\beta\ell$	The amplitude of position modulation
b_s	The instantaneous value of stub susceptance, normalized units
$\overline{b_s}$	The average value of stub susceptance, time-average of b_s; normalized units

Δb The amplitude of susceptance modulation

$\overline{K_L}$ The complex load reflection coefficient

j The imaginary operator

y_1 The admittance seen looking into the transmission line toward the load at the stub position, not including the stub susceptance; the line admittance; normalized units

K The magnitude of a reflection coefficient

g_1 The conductance component of the line admittance, normalized units

y_t The total input admittance at the stub position; the sum of the line admittance and the stub susceptance; normalized units

b_1 The susceptance component of the line admittance; normalized units

K_L The magnitude of the load reflection coefficient

$\overline{K_a}$ The complex reflection coefficient at the position of a voltage minimum on the transmission line

y_s The admittance of the matching stub, normalized units

βl_1 The electrical distance from the load to the position of a voltage minimum

θ The electrical distance from the position of the voltage minimum to the position of the matching stub

φ_L The phase angle of the load reflection coefficient

y_a The input admittance of the transmission line at the position of a voltage minimum, looking toward the load; normalized units

y_θ The input admittance of the transmission line at an arbitrary position an electrical distance from the position of a voltage minimum; normalized units

$\overline{K_t}$ The "total reflection coefficient"; the complex reflection coefficient at the position of the matching stub

K_t The magnitude of the total reflection coefficient

$\overline{\theta}$ The average electrical distance θ from the position of a voltage minimum on the transmission line to the stub position; the time-average value of θ

$\Delta\theta$ The amplitude of position modulation, equal to $\Delta\beta l$

K_{t_M} The magnitude of the total reflection coefficient when the stub position and susceptance have met the conditions for a match

K_{t_o} The non time varying component of the total reflection coefficient

K_{t_1} The time varying component of the total reflection coefficient which varies at the modulation frequency

K_{t_2} The time varying component of the total reflection coefficient which varies at twice the modulation frequency

db Decibel; equal in magnitude to twenty times the

common logarythm of the ratio of output voltage to input voltage

CHAPTER I

INTRODUCTION

1. Definition of the term "match".

The term "match" has been used in two different senses, both applicable to systems including transmission lines; and it is necessary to differentiate between these two senses and to define the type of match to be effected by the automatic system.

In one sense, the term "match" is associated with a transmission line so terminated that no reflection takes place at the termination; this requires that the admittance of the termination be equal to the characteristic admittance of the transmission line. This condition might better be described by the use of the term "equal match".

In a different sense, the term "match" is associated with maximum transfer of power from a generator to the admittance at the generator terminals; this requires that the admittance seen at the generator terminals be the complex conjugate of the internal admittance of the generator. This condition might better be described by the use of the term "conjugate match".

In a system utilizing transmission lines which are an appreciable part of a wavelength long, it is desirable that there be no standing waves on the transmission line; that is, that there be no reflection at the terminating admittance. Such a condition insures that all the power being transmit-

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ted by the transmission line is delivered to the load and also insures that the voltage across the transmission line will not approach the breakdown value at any point along the line (unless, of course, the input power level is too high). Thus it is apparent that at the termination of the transmission line, an "equal match" is desired, since only this type of match will eliminate reflection at the terminating admittance.

On the other hand, at the input terminals of the transmission line (which are the output terminals of the generator), maximum transfer of power from the generator to the transmission line is desired. At the input terminals, then, the "conjugate match" is desired.

If the transmission line is lossless, its characteristic admittance is real, and the two senses of the term "match" become identical, since the conjugate of a real number is identically equal to the number itself. Thus in a system in which the transmission line is assumed to be lossless, both the terminating admittance and the internal admittance of the generator should be equal to the characteristic admittance of the transmission line. It is the usual practice to consider the transmission line lossless for purposes of design. The loss may then be taken into consideration after design of the system; that is, the loss of the transmission line is considered to be important only in its effect on the attenuation of the signal, and not in its effect on the char-

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acteristic admittance of the line.

Most microwave generators have included in the body of the generator a matching transformer, such as the matching iris used in a magnetron, to provide the required transformation from the internal admittance of the generator to the characteristic admittance of the transmission line. It is then the purpose of the designer of the transmission system to provide the "equal match" at the termination of the transmission line.

It is apparent then that if the automatic matching device is to be used to match the load to the transmission line; that is, to eliminate standing waves on the transmission line, the device is concerned with the "equal match" defined above. The function of the automatic impedance matching system is to provide an admittance equal to the characteristic admittance of the transmission line at some point between the load admittance and the generator.

2. The need for an automatic matching system.

It is generally desirable that a generator of microwave signals operate into a matched load. This is particularly true of the magnetrons which are used in most of the present day moderate and high powered systems. The magnetron is characterized by high sensitivity to changes in load admittance. Small variations in load admittance may result in significant changes in frequency of oscillation and may cause instability in some ranges of load admittance. Indeed, the

importance of the Reike diagram and the necessity for its use in microwave design are due to this sensitivity of the magnetron to variations in load admittance. The effects of variation in load admittance can be eliminated, however, by providing a matching device in the transmission system to transform the admittance seen at that point in the transmission system to the characteristic admittance of the transmission line. If this device were able to follow the variations of load admittance caused by rotating devices, etc., the magnetron would always see as its terminating admittance the characteristic admittance of the transmission line.

In moderate and high powered systems using magnetron oscillators there is a more serious result of an unmatched load: The power reflected from the load to the magnetron may cause burnout of such components as matching irises, thus leading to failure of the entire system. Again, the provision for an automatic equal match would eliminate this cause of failure by eliminating the reflected power.

In many applications the ability to change rapidly the frequency of operation of a system would be desirable. But while tunable magnetrons are available, changing the frequency of the system by a significant amount usually also entails retuning (manually) many other components in the system in order to maintain the voltage standing wave ratio (VSWR) within allowable limits, since many of the components in the transmission system are frequency sensitive. Among these frequen-

cy sensitive components are rotating joints, choke joints, and the radiating element which constitutes the terminating load. The bandwidths over which these components have an acceptably low VSWR are sometimes quite narrow, and may be increased only by costly and time-consuming development. Again the use of an automatic matching device would permit relatively large changes in the frequency of operation, since the values of VSWR introduced by the frequency sensitive components would be "matched out" by the matching device.

Thus it is apparent that the incorporation of an automatic matching system would ease the design requirements of many components, facilitate rapid frequency changes of relatively large magnitude, provide more stable frequency of oscillation and power output from the magnetron, and protect the magnetron from excessive amounts of reflected power; and would therefore provide more efficient operation of the entire system.

3. Desirable matching system characteristics.

The requirements which an automatic matching system should meet are:

(a) Provide a very low VSWR even when the load reflection coefficient is moderately high, say 0.6, corresponding to a load VSWR of 4.0.

(b) Operate with sufficient speed to follow such load variations as are introduced by rotating joints in a search radar.

(c) Cause little attenuation in the desired transmission direction; that is, have low loss.

(d) Maintain the above characteristics over a relatively broad frequency band.

4. Selection of a general type of matching device.

Several manually operated matching devices are in use, both in operating systems and in the laboratory. These include:

(a) Single movable shunt susceptance,

(b) double fixed position shunt susceptances, and

(c) triple fixed position shunt susceptances.

These devices may also be made up in series form, although this is usually not the case due to the difficulty of adjustment of the series forms. In different types of transmission lines these devices take different forms. For example, in parallel wire transmission lines, these types may be made up of one or more stubs with either open or short circuit terminations; in waveguide, they may be made up of one or more capacitive or inductive screws, or one or more short circuited E-plane or H-plane shunt arms.

An ideal single movable shunt susceptance is capable of matching any load. The required adjustments are two: The electrical position of the shunt and the value of the shunt susceptance. It is frequency sensitive in that the required physical position to match a given load changes as the frequency changes. The value of susceptance required to match a given

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load is not frequency sensitive, although the method used to obtain this value of susceptance normally introduces frequency sensitivity. For example, the parallel wire transmission line shunt stub, in which the length of stub which produces a given value of susceptance is a function of the frequency of operation.

The double fixed position shunt susceptances are capable of matching only a certain range of values of load admittance, this range of values being determined by the electrical spacing of the two shunts. The two required adjustments are the values of the two shunt susceptances. The range of admittances which can be matched is a function of frequency, since the electrical separation of the two shunts is a function of frequency; and the device is also frequency sensitive in the same manner as the single shunt susceptance with respect to the values of susceptances required to match a given load at different frequencies.

An ideal triple shunt susceptance is capable of matching any load, but three adjustments are required: The values of the three shunt susceptances. These adjustments are interrelated to a degree which makes even manual adjustment difficult.

In view of the characteristics of the basic types of matching devices, the single movable shunt susceptance seems most easily adapted to an automatic matching system, because it is capable of matching any load and requires only two ad-

justments. In a parallel wire transmission line this type takes the form of a single movable shunt stub of variable length with either an open or a short circuit termination. Since a true open circuit termination is impossible to achieve, the short circuit termination is normally used. Adjustment of the length of this stub by varying the position of the short adjusts the value of the susceptance. In waveguide transmission systems, this type usually takes the form of a capacitive or inductive probe of variable position and depth of penetration into the waveguide; but may also take the form of a short circuited E-plane or H-plane shunt arm of variable length and variable electrical position. In the case of the shunt arms, the position is usually varied by the use of a variable phase shifter between the load and the shunt arm.

CHAPTER II

THE SCHEME OF OPERATION

1. The admittance characteristics of a transmission line.

The input admittance of a transmission line of arbitrary length and terminated in an arbitrary value of load admittance depends on the electrical length of line between the input and the termination and on the value of the terminating admittance, and is periodic; that is, admittance values are repeated each half wavelength along the line. If the terminating admittance is equal to the characteristic admittance of the line, the input admittance of the line is equal to the characteristic admittance of the line and is independent of the length of the line, there is no reflection from the termination, and there is no standing wave on the line. If the terminating admittance is not equal to the characteristic admittance of the line there is a reflection at the termination, a standing wave exists on the line, and the input admittance is dependent on the length of the line and the value of the terminating admittance. This latter case is the mismatched case with which the matching system must deal.

Within each half wavelength along the mismatched line there are two points at which the input conductance is equal to the conductance component of the characteristic admittance of the line. It will be assumed that the transmission line is lossless. Therefore, the characteristic admittance of the

line is real, and at the aforementioned points the input conductance of the line is equal to the characteristic admittance of the line. At these points the magnitude of the input susceptance depends on the magnitude of the load reflection coefficient, and is positive at one of the points and negative at the other. If at either of these points a susceptance equal in magnitude and opposite in sign to the input susceptance of the line is placed in parallel with the line, the total input admittance at this point becomes equal to the characteristic admittance of the line and no reflection will take place at this point. Then the line is matched at this point and there will be no standing wave along the line at distances from the load greater than that at which the shunt susceptance has been placed, although a standing wave still exists between the shunt susceptance and the load.

As described above, there are two points within each half wavelength of the mismatched line at which a match may be attained. At one of these points a positive shunt susceptance will be required, and at the other, a negative shunt susceptance will be required. Since a match may be attained at either of these two points, the device used to provide the shunt susceptance need not be capable of all values of susceptance, but rather either all positive values of susceptance or all negative values of susceptance. It will be shown later that in the practical case where the magnitude of the load reflection coefficient may be somewhat

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restricted in an actual system, the normalized magnitude of the shunt susceptance will range from zero to, say, two. Normalized admittances will be used throughout this paper for simplicity.

2. The single movable shunt susceptance tuner.

In Chapter I, the single movable shunt susceptance was chosen for use in the automatic matching system because of its desirable characteristic of having only two required adjustments and its capability of matching any load. The admittance characteristics of this device should be more fully examined.

If a section of lossless transmission line is terminated in either an open circuit or a short circuit, the input admittance has the following characteristics:

(a) The conductance component is zero regardless of the length of the section of line, and

(b) the susceptance component takes on all values of susceptance from minus infinity to plus infinity and is a function of the length of the section of line.

Since a true open circuit termination is impossible to attain, the short circuit termination is usually used. Figure 1 shows the input susceptance, as a function of electrical length of line, of a section of transmission line terminated in a short circuit. The input conductance is always zero since the line is assumed to be lossless. The input susceptance is negative and infinite at a length of zero (this

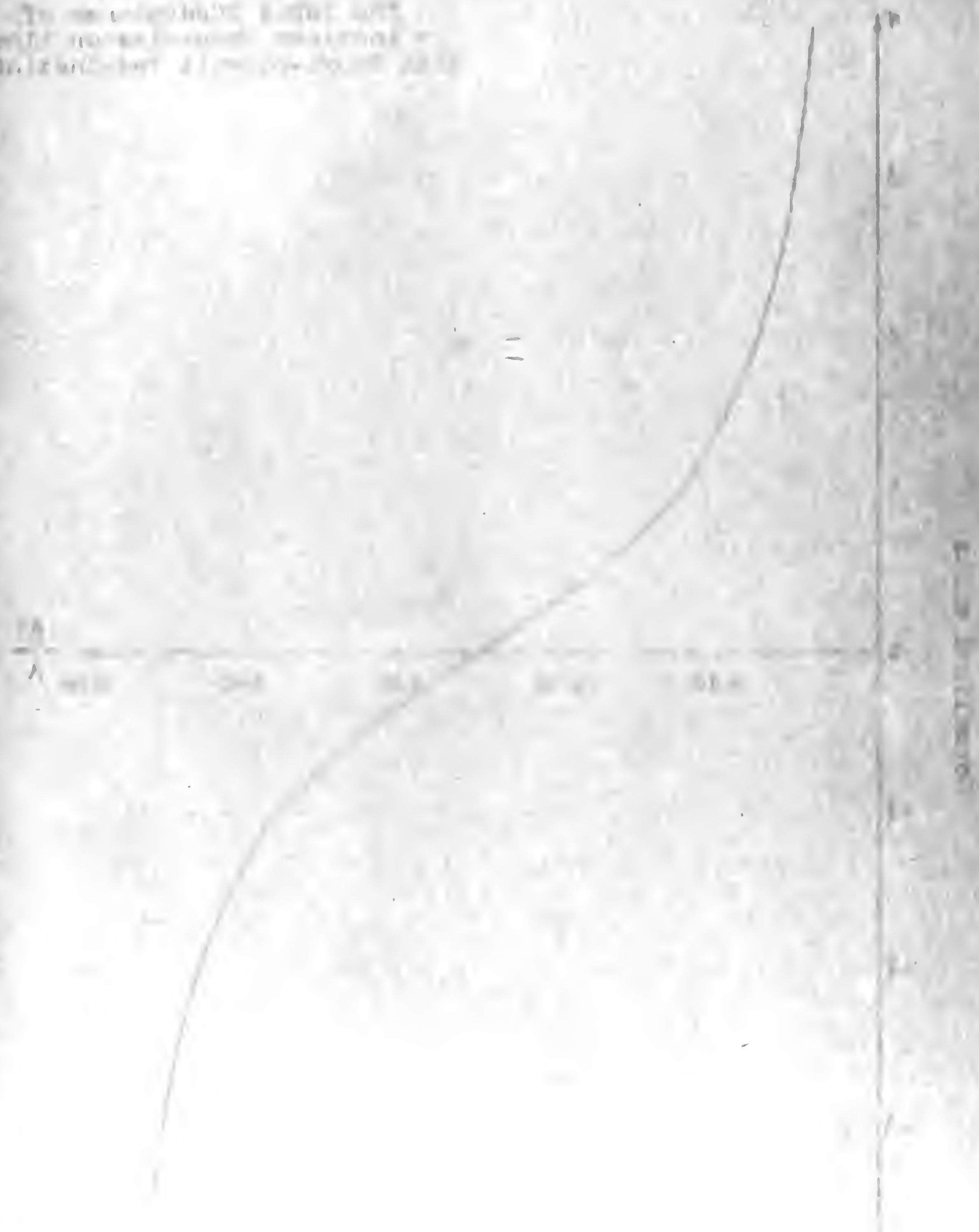
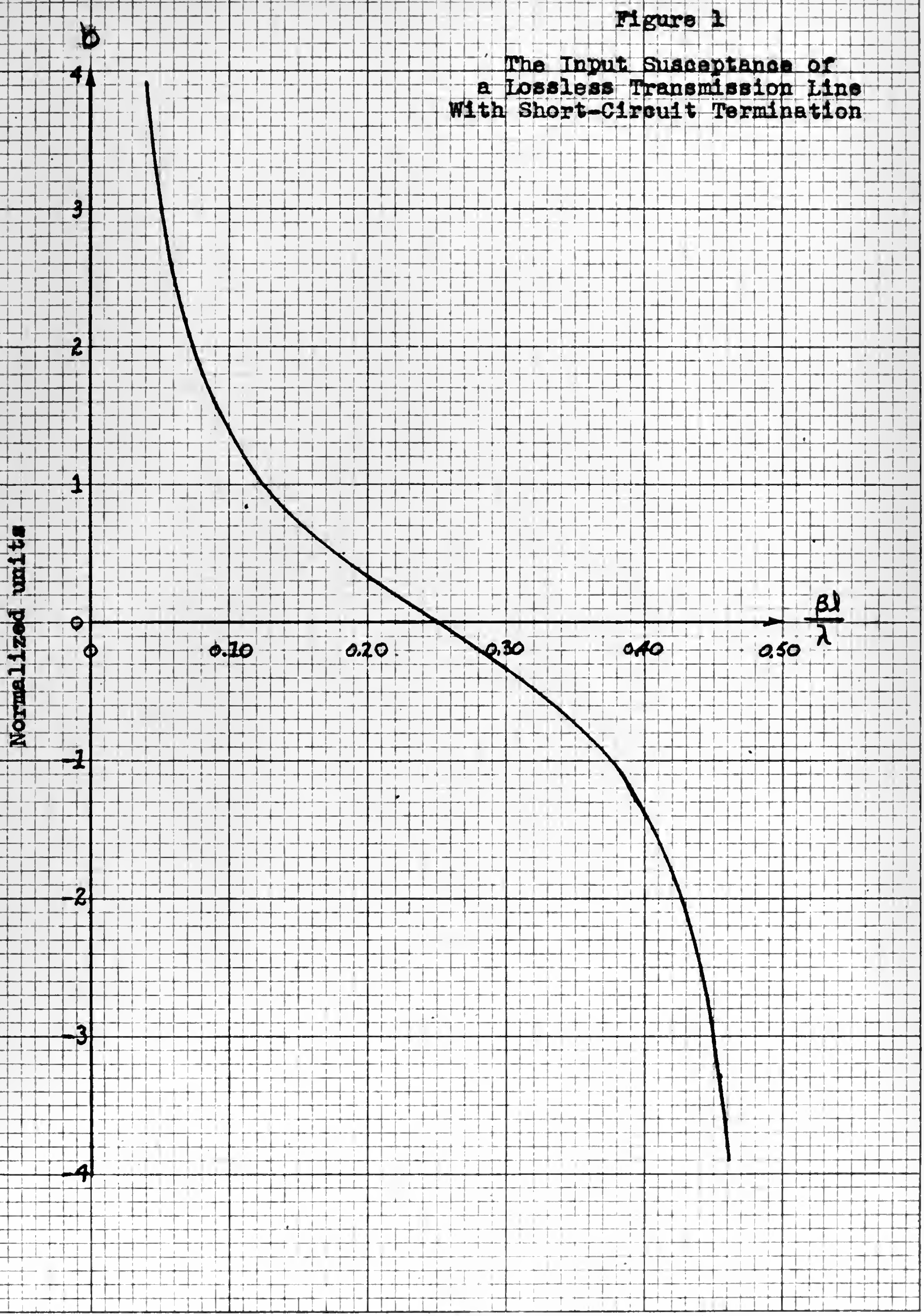


Figure 1

The Input Susceptance of
a Lossless Transmission Line
With Short-Circuit Termination



might be taken as an infinite conductance), is negative for lengths between zero and one quarter wavelength, is zero at a length of exactly one quarter wavelength, is positive for lengths between one quarter wavelength and one half wavelength, and is positive and infinite at a length of exactly one half wavelength (this again might be taken as an infinite conductance). Thus as the electrical length is varied from zero to one half wavelength, all values of susceptance are provided at the input of the short circuited section of line.

3. Matching a line with the single shunt susceptance.

The following description of the use of the single shunt susceptance tuner points out the method of determining the stub position and the value of the stub susceptance.

The value of the terminating admittance is determined by one of the several available methods of measurement. Then, using a Smith Chart or other circle diagram, the electrical positions on the transmission line at which the input conductance is unity are determined. The magnitude of the input susceptance is the same at each of these points, and is negative at one and positive at the other. If the point at which the input susceptance is negative is chosen for the stub position, then the stub susceptance must be positive and equal in magnitude to the line input susceptance. The length of a shorted stub which provides this value of shunt susceptance may then be determined from a graph such as Figure 1, or by use of the Smith Chart. If the point at which the line input

[illegible]

susceptance is positive is chosen for the stub position, then the stub susceptance must be negative and equal in magnitude to the line input susceptance.

The required position of the stub and the required value of its susceptance may also be computed from admittance transformation formulae derived in Chapter IV, but such computations are laborious.

If the magnitude of the reflected wave can be observed, (perhaps by the use of a directional coupler and meter), the stub position and length can be correctly set quite easily by alternately adjusting the position and length of the stub to successively minimize the magnitude of the reflected wave. Although these adjustments are not independent, they are simultaneously convergent; that is, successive adjustment of the position and length alternately will ultimately lead to the matched condition; minimizing the reflected wave by making one adjustment does not introduce a false error indication in the other adjustment. It is this simultaneous convergence which makes the stub easily adapted to automatic operation.

4. The development of signals - introduction.

It was pointed out above that the stub might be correctly positioned and adjusted in length by observing the magnitude of the reflected wave. In making adjustments in this manner, directional sense is provided by the meter and observer; that is, the observer makes an adjustment and observes whether a reduction in the magnitude of the reflected wave

results. If a reduction takes place, the observer continues the adjustment in the same direction until a minimum is reached; if a reduction does not result, the observer reverses the direction of adjustment and adjusts for a minimum. This implies that the reflected wave may be used to generate signals for use in servo loops controlling the position and length of the stub. However, if the initial motions are to be in the correct direction to provide a match, some means of providing immediate directional sense must be included.

In the discussion of the manual adjustment of the tuner, it was pointed out that a directional coupler and meter can be used to adjust the tuner to match by observing the magnitude of the reflected wave. The magnitude of the reflected wave is simply its amplitude and is detected by some sort of rectifying device such as a crystal detector. Since the amplitude of the reflected wave plays an essential role in the manual adjustment of the tuner, it seems desirable to examine the variations which take place in the amplitude of the reflected wave as the stub is adjusted from the position of match to positions of mismatch on either side of the position of match; and as the stub susceptance is adjusted from the value of match to values greater than, and less than, the value of match. If this is done in a qualitative manner, the variations which take place are intuitively apparent.

Consider the system shown in Figure 2, consisting of a transmission line, a load admittance not equal to the char-

acteristic admittance of the transmission line, and a single movable tuning stub. The position of match is designated by M and the match value of the stub susceptance is designated by b_{s_M}.

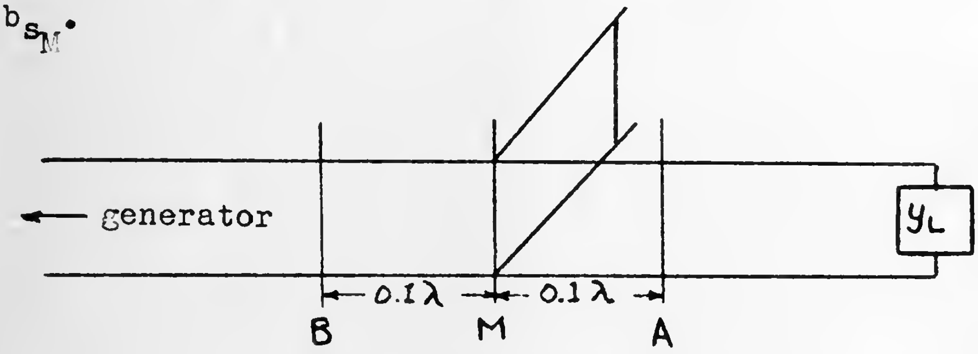


Figure 2.

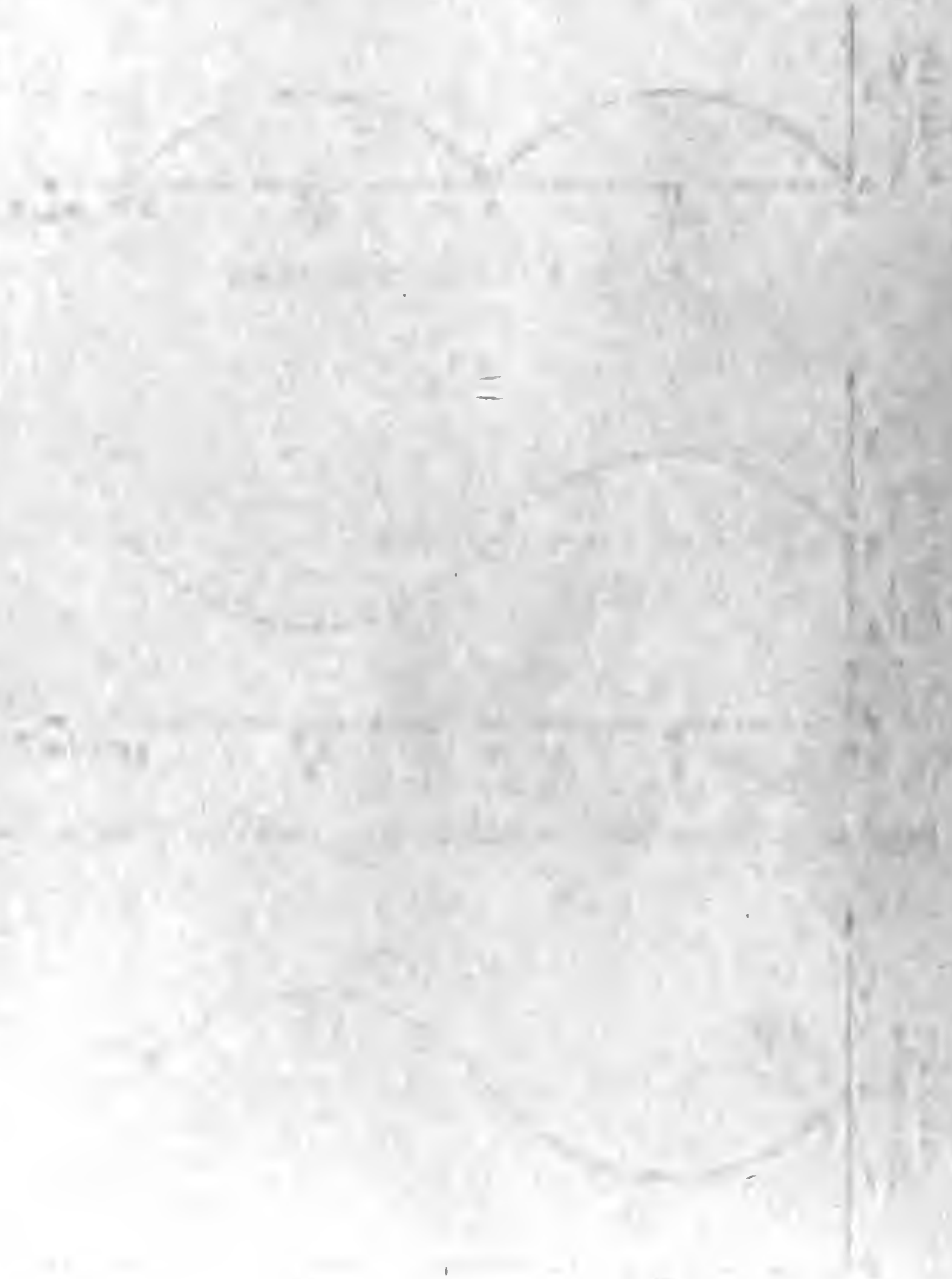
The Transmission System

5. The development of position signals.

Consider, as the first case, that the stub is at the position of match and that the stub susceptance has the value of match; and examine the effects of the position of the stub. Under the conditions specified above, the amplitude of the reflected wave is zero, since the conditions for a match at the stub position are met. If the stub is moved a short distance in either direction from the position of match, the amplitude of the reflected wave will increase. If the stub is moved alternately a short distance, say 0.05λ, (where λ is the wavelength along the transmission line) to either side of the position of match with simple harmonic motion of angular frequency ω_m, the reflected wave will be amplitude modulated at an angular frequency $2\omega_m$, passing through zero each time

the stub passes through the position of match, and reaching a maximum amplitude at the maximum excursions of the stub from the position of match. The resulting variation in amplitude of the reflected wave as a function of $\omega_m t$ (where t represents time) is shown in Figure 3. It is assumed that the stub starts from the position of match and moves initially toward the load. The exact shape of the waveform of Figure 3 is not yet determined; it is assumed to be a rectified sine wave.

As the second case, consider that the stub susceptance still has the value of match, but that the stub position is a short distance toward the load from the position of match. Assume that the stub position is that of point A in Figure 2, 0.10λ from the position of match toward the load. Under these conditions the amplitude of the reflected wave is <u>not</u> zero. If the stub is moved away from the position of match; that is, toward the load, the amplitude of the reflected wave is increased. If the stub is moved toward the position of match; that is, toward the generator, the amplitude of the reflected wave is decreased. If the stub is moved alternately toward the load and toward the generator with simple harmonic motion about point A with angular frequency ω_m, and again with maximum excursion of 0.05λ, the resulting variation in the amplitude of the reflected wave will be as shown in Figure 4. Note that in this case the stub starts from point A of Figure 2, and again moves initially toward the



Amplitude Variations of the Reflected Wave
Due to Position Modulation

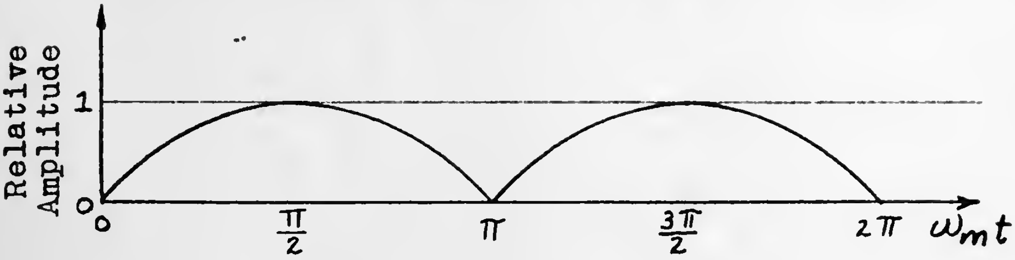


Figure 3 Match Conditions

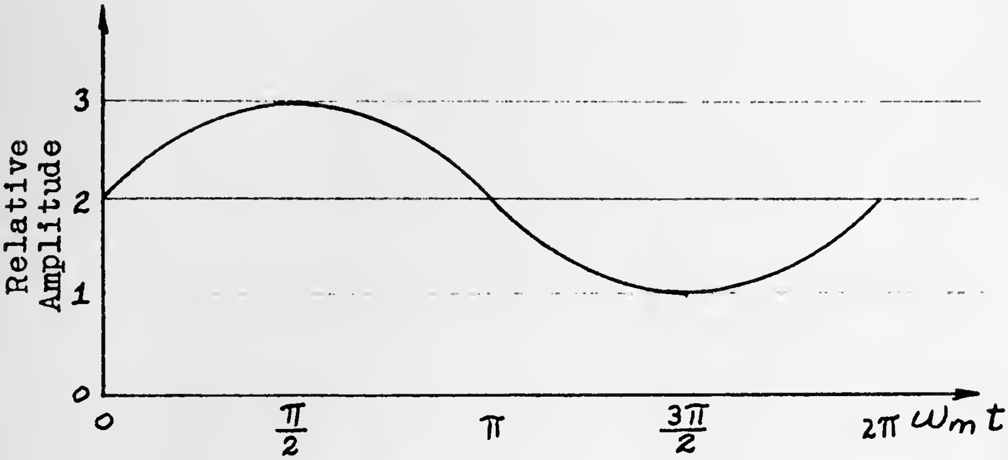


Figure 4 Match Susceptance; Stub Position Toward Load

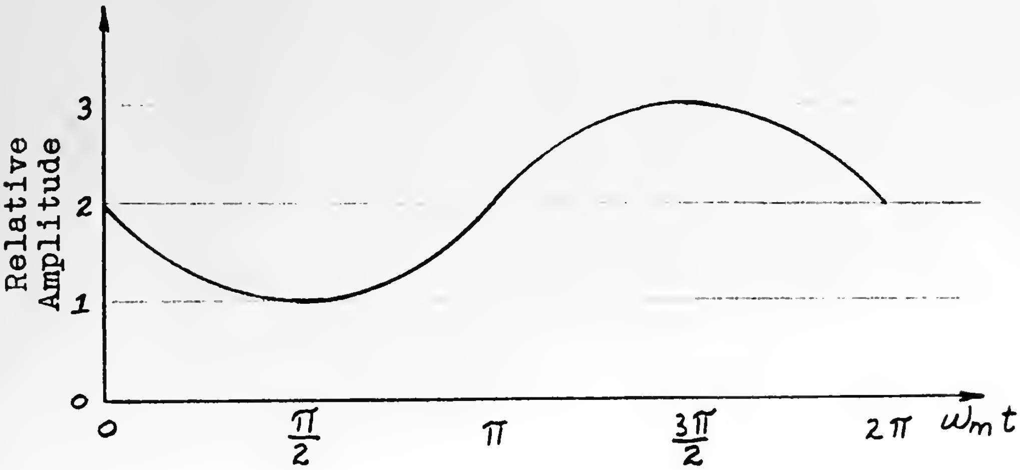


Figure 5

Match Susceptance; Stub Position Toward Generator

load. The exact shape of the waveform of Figure 4 has not yet been determined; it is assumed to be sinusoidal.

As the third case, consider that the stub susceptance again has the value of match, but that the stub position is a short distance toward the generator from the position of match. Assume that the stub position is that of point B in Figure 2, 0.10λ from the position of match toward the generator. Under these conditions the amplitude of the reflected wave is again <u>not</u> zero. If the stub is moved toward the position of match; that is, toward the load, the amplitude of the reflected wave is decreased. If the stub is moved away from the position of match; that is, toward the generator, the amplitude of the reflected wave is increased. If the stub is moved alternately toward the load and toward the generator with simple harmonic motion about point B with angular frequency ω_m, and again with maximum excursion of 0.05λ, the resulting variation in the amplitude of the reflected wave will be as shown in Figure 5. Note that in this case the stub starts from point B of Figure 2, and again moves initially toward the load. The exact shape of the waveform of Figure 5 has not yet been determined; it is assumed to be sinusoidal.

Now examine the waveforms of Figures 3, 4, and 5. Note that in each case the initial motion of the stub has been toward the load, and in each case the angular frequency of the simple harmonic motion of the stub has been ω_m, which

will be termed the modulation frequency. Note that when the initial position of the stub is on the load side of the match position (point A of Figure 2) the variation of the amplitude of the reflected wave is a sine wave of angular frequency ω_m and zero phase angle (Figure 4). Note that when the initial position of the stub is on the generator side of the match position (point B of Figure 2) the variation of the amplitude of the reflected wave is a sine wave of angular frequency ω_m and 180 degree phase angle (Figure 5). Note that when the initial position of the stub is the match position (point M of Figure 2) the variation of the amplitude of the reflected wave occurs <u>not</u> at the modulation frequency, ω_m, but at twice the modulation frequency (Figure 3).

Now if the stub position can be continuously varied with simple harmonic motion at the modulation frequency, it is apparent that the reflected wave will be continuously amplitude modulated. The signal represented by this amplitude modulation of the reflected wave is zero when the stub is at the match position (that is, there is no modulation of the reflected wave at the modulation frequency), and reverses phase when the stub passes through the position of match. Therefore, modulating the position of the stub will generate a signal which contains directional sense and which becomes zero when the stub is at the position of match. To move the stub to the position of match it is only necessary to apply this signal to an appropriate servo loop controlling the position of

the stub. It should be noted that at the position of match the amplitude of the reflected wave has an average value. This indicates that a perfect match cannot be obtained by this method; however, this does not destroy the usefulness of the method since the automatic matching device is not required to provide a perfect match, but rather to provide an acceptable value of VSWR. It will be shown later that this average value of the reflected wave can be made quite small, corresponding to a VSWR of about 1.05.

6. The development of susceptance signals.

Now apply the same method of qualitative examination to the value of the stub susceptance. As the first case, consider that the stub susceptance has the value of match and that the stub is at the position of match, and examine the effect of variations in stub susceptance on the amplitude of the reflected wave. Under the conditions specified above, the amplitude of the reflected wave is zero, since the conditions for a match at the stub position are met. If the stub susceptance is either increased or decreased from the match value, the amplitude of the reflected wave will be increased. If the stub susceptance is alternately increased and decreased a small amount, say 0.05 (normalized), from the match value with simple harmonic variation at the modulation frequency, ω_m, the reflected wave will be amplitude modulated at twice the modulation frequency, passing through zero each time the stub susceptance passes through the match value, and

reaching a maximum amplitude at the maximum excursions of the stub susceptance from the match value. The resulting variation in the amplitude of the reflected wave as a function of $\omega_m t$ is shown in Figure 6. It is assumed that the stub susceptance starts at the match value and initially increases. The exact shape of the waveform of Figure 6 is not yet determined; it is assumed to be a rectified sine wave.

As the second case, consider that the stub is still at the position of match, but that the stub susceptance is greater than the match value. Under these conditions the amplitude of the reflected wave is not zero. If the stub susceptance is increased, the amplitude of the reflected wave is increased. If the stub susceptance is decreased, the amplitude of the reflected wave is decreased. If the stub susceptance is alternately increased and decreased with simple harmonic variation at the modulation frequency about the initial value with the same maximum excursion as in case one above, the variation in amplitude of the reflected wave will be as shown in Figure 7. The exact shape of the waveform of Figure 7 is not yet determined; it is assumed to be sinusoidal.

As the third case, consider that the stub is still at the position of match, but that the stub susceptance is less than the match value. Under these conditions the amplitude of the reflected wave is again not zero. If the stub susceptance is increased, the amplitude of the reflected wave is decreased. If the stub susceptance is decreased, the ampli-

Amplitude Variations of the Reflected Wave
Due to Susceptance Modulation

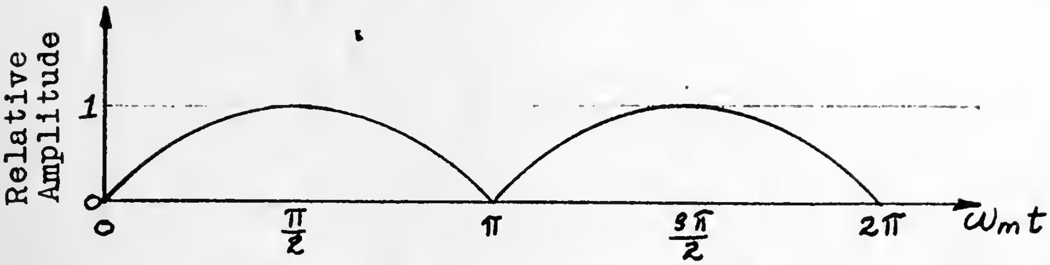


Figure 6 Match Conditions

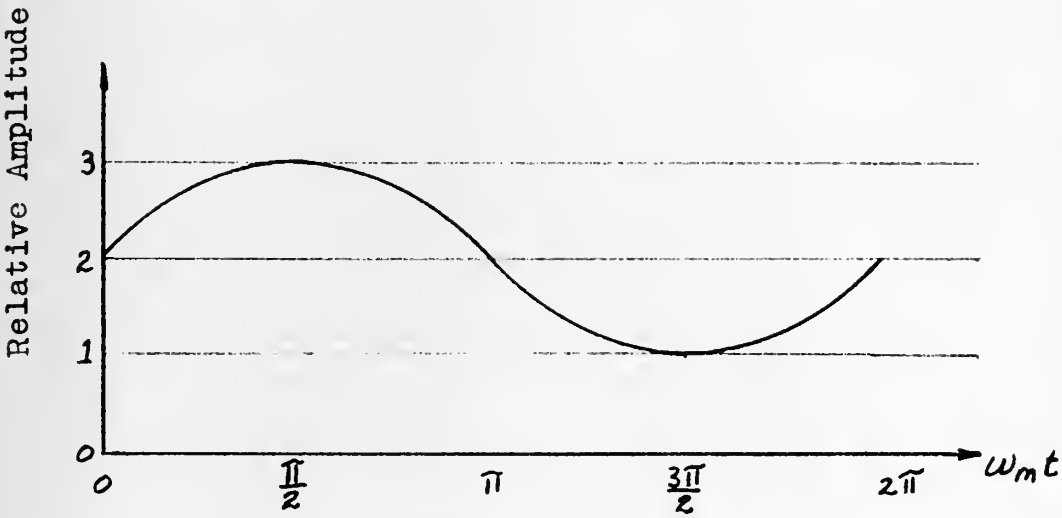


Figure 7

Stub Susceptance Greater Than Match Value; Match Position

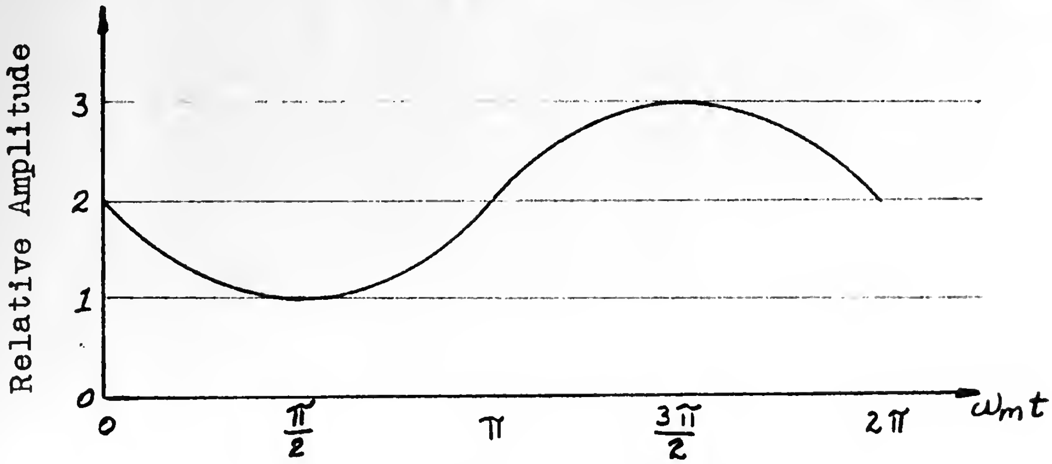


Figure 8

Stub Susceptance Less Than Match Value; Match Position

[illegible]

tude of the reflected wave is increased. If the stub susceptance is alternately increased and decreased with simple harmonic variation at the modulation frequency about the initial value with the same maximum excursion as in cases one and two above, the variation in amplitude of the reflected wave will be as shown in Figure 8. The exact shape of the waveform of Figure 8 is not yet determined; it is assumed to be sinusoidal.

Now examine the waveforms of Figures 6, 7, and 8. Note that in each case the initial variation in stub susceptance has been toward larger values, and in each case the angular frequency of the simple harmonic variation of stub susceptance has been ω_m, the modulation frequency. Note that when the initial value of stub susceptance is greater than the match value, the variation of the amplitude of the reflected wave is a sine wave of angular frequency ω_m and zero phase angle (Figure 7). Note that when the initial value of stub susceptance is less than the match value, the variation of the amplitude of the reflected wave is a sine wave of angular frequency ω_m and 180 degrees phase angle (Figure 8). Note that when the initial value of stub susceptance is equal to the match value, the variation of the amplitude of the reflected wave occurs <u>not</u> at the modulation frequency, but at twice the modulation frequency (Figure 6).

Now if the stub susceptance can be continuously varied with simple harmonic variation at the modulation frequency,

it is apparent that the reflected wave will be continuously amplitude modulated. The signal represented by this amplitude modulation of the reflected wave is zero when the stub susceptance is equal to the match value (that is, there is no modulation of the reflected wave at the modulation frequency), and reverses phase when the stub susceptance passes through the match value. Therefore, modulating the value of the stub susceptance will generate a signal which contains directional sense and which becomes zero when the stub susceptance is equal to the match value. The stub susceptance may be modulated by modulating the length of the stub; and if the amplitude of susceptance modulation is small, such modulation will be very nearly linear. To obtain the match value of stub susceptance it is only necessary to apply the signal developed by the modulation of the stub susceptance to an appropriate servo loop controlling the length, and therefore the susceptance, of the stub. It should be noted that at the match value of stub susceptance the amplitude of the reflected wave has an average value. This again indicates that a perfect match cannot be obtained by this method; however this does not destroy the usefulness of the method for the reasons which were pointed out in the discussion of the position modulation.

7. Position and susceptance modulation considerations.

A system of amplitude modulation of the reflected wave to provide signals containing directional sense has been pro-

posed by Walters [14] and investigated by Red [11]. The proposed system generates the position and susceptance signals in a manner similar to, but not identical with, the method discussed above in sections 5 and 6.

The methods discussed in sections 5 and 6 consist essentially of sinusoidally varying the position of the stub through a small amplitude to achieve an amplitude modulation of the reflected wave containing directional sense, and sinusoidally varying the length of the stub through a small amplitude to achieve susceptance modulation which results in a second amplitude modulation of the reflected wave containing directional sense. If these signals or modulation components of the reflected wave are to be used to adjust the position and length of the stub they must be separable in some manner in order to be applied to the two required servo loops. Two simple methods of providing this separability are available:

(a) The two modulations may be accomplished at different frequencies and separated by filters in the servo amplifiers, or

(b) the two modulations may be at the same frequency, but of different phases, and the signals separated by phase sensitive detectors.

The latter method was proposed by Red [11], and was used in the system described in Chapter V. The required phase difference may be provided mechanically in the modulation drive

system; for optimum isolation between the two servo loops, the phase difference should be 90 degrees. Each phase sensitive detector will require a reference phase; these may be provided by driving a two phase spin generator from the modulation drive motor. In connection with the modulation drive system, it should be pointed out that the position modulation proposed does not necessarily imply mechanical motion of the stub, since it is only necessary to vary the electrical position of the stub on the line. In fact, in the system discussed in Chapter V, the mechanical position of the susceptance probe, which is analagous to the stub here being discussed, is not modulated; rather, the electrical separation of probe and load is varied by the use of a variable phase shifting device.

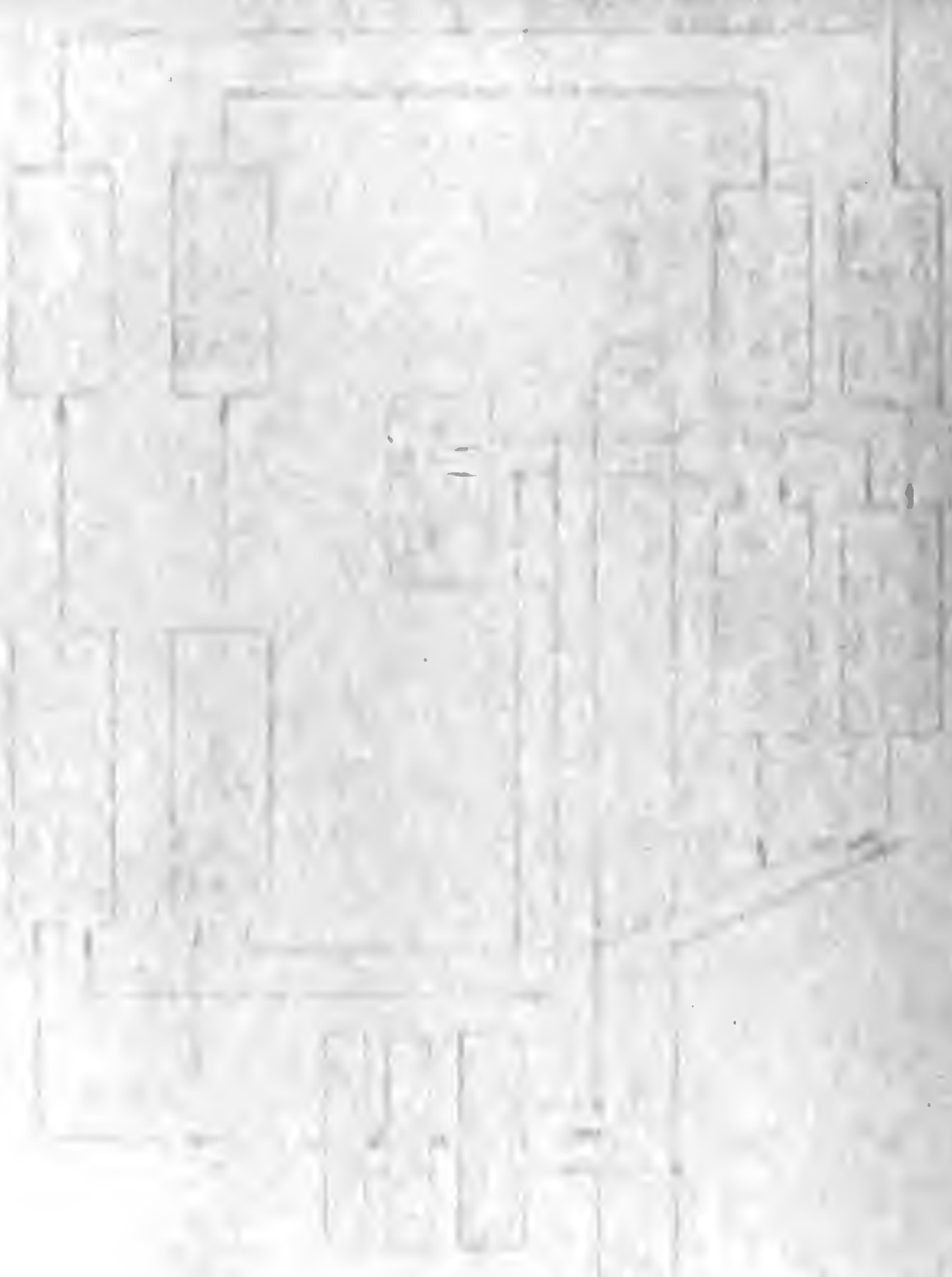
8. Detection of the reflected wave.

Any one of the usual microwave or ultra-high frequency rectifying detectors may be used to detect the reflected wave and thus extract the signals contained in the modulation of the reflected wave. The type of detector used will of course affect the waveform provided at the output terminals, but due to the presence of the average amplitude of the reflected wave, even the use of a square law detector, such as a crystal, will not result in the loss of the fundamental modulation frequency signals. In fact, in the mathematical analysis of Chapter IV, it will be assumed that a square law detector is used, since this assumption results

in a considerable simplification of the analysis. It should be noted here that a square law detector, a silicon crystal diode, was actually used in the system presented in Chapter V.

9. The complete system in general terms.

A qualitative discussion of the entire system has now been presented, and it is possible to lay out a general plan for the entire system. The elements of the system are the modulation drive system and reference phase generator, the detector, a preamplifier, two phase sensitive rectifiers to separate the two signals, a power amplifier associated with each servo loop, and the position and susceptance drive mechanisms. The entire system is shown in the block diagram of Figure 9.



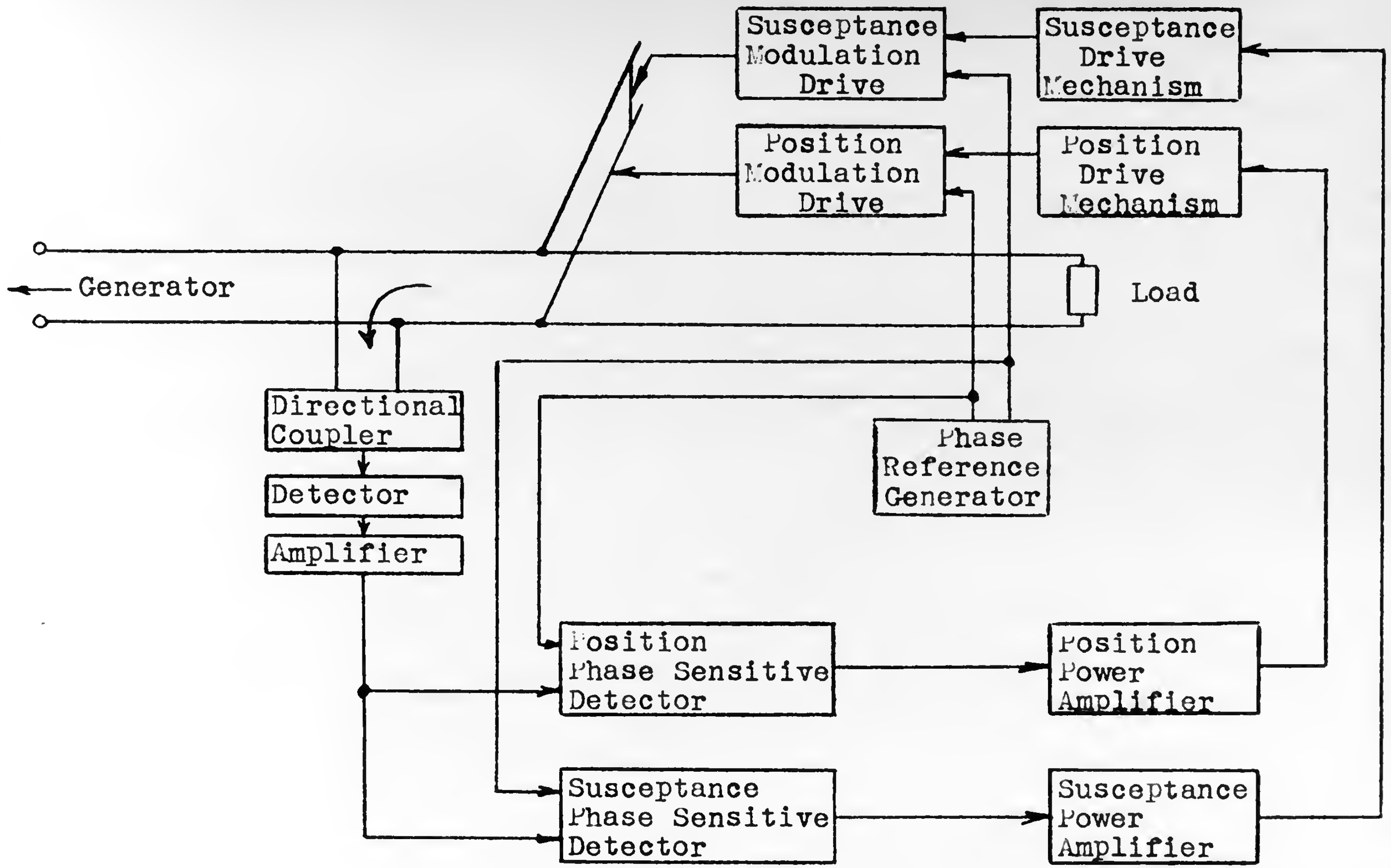


Figure 9

Block Diagram of an Automatic Impedance Matching System

CHAPTER III

A QUALITATIVE ANALYSIS OF THE POSITION AND SUSCEPTANCE MODULATIONS

1. Introduction.

In Chapter II an explanation of the methods which may be used to generate position and susceptance signals has been given. In this chapter, a qualitative analysis of the position and susceptance modulations and their effect on the reflection coefficient at the stub position will be given. The two modulations will be examined separately, as though they were completely independent. An examination of the possibility of cross-talk between the two modulations is included at the end of the mathematical analysis of Chapter IV.

It will be assumed that the incident wave in the transmission system is constant, so that the reflected wave is directly proportional to the reflection coefficient at the stub position. Thus an analysis of the variations in the magnitude of the reflection coefficient yields directly the amplitude variations of the reflected wave. In the following analysis the terms "an increase in reflection coefficient" and "a decrease in reflection coefficient" will be used frequently. These terms are here defined to mean an increase or a decrease in the magnitude of the reflection coefficient at the stub position.

2. Definitions of the position and susceptance modulations.

[illegible]

The position of the stub on the transmission line, including the effect of the position modulations, may now be defined by

$$\beta l = \overline{\beta l} + \Delta\beta l \sin \omega_m t. \qquad 3\text{-}1$$

β is the imaginary part of the complex propagation constant, γ. The units of β are radians or degrees per unit length. (Since the transmission line is assumed to be lossless, the real part of the complex propagation constant, the attenuation per unit length, is zero). l is physical length along the transmission line; so that, in general, βl is electrical length along the transmission line. In equation 3-1, βl is the instantaneous electrical separation of the stub from the load, and is defined as being positive when measured from the load to the stub; that is, in the direction from the load toward the generator. $\overline{\beta l}$ is the average electrical distance from the load to the stub, the time- average value of βl. $\Delta\beta l$ is the amplitude of the position modulation. ω_m is the angular modulation frequency.

Similarly, the stub susceptance may be defined by

$$b_s = \overline{b_s} + \Delta b \cos \omega_m t. \qquad 3\text{-}2$$

b_s is the instantaeous value of stub susceptance in normalized units. $\overline{b_s}$ is the average value of stub susceptance, the time-average value of b_s. Δb is the amplitude of susceptance modulation in normalized units. ω_m is again the angular mod-

ulation frequency.

It should be noted that the two modulations are 90 degrees out of phase at the modulation frequency, as was proposed in Chapter II. Equation 3-2 involves an approximation: In Chapter II it was proposed that the susceptance modulation be achieved by modulating the length of the stub. It has been assumed that if the amplitude of susceptance modulation is small, the variation of susceptance with variation in stub length is very nearly linear. This is a good approximation if the amplitude of susceptance modulation is 0.1 normalized units or less, as may be seen by reference to Figure 1. However, it is also apparent from Figure 1 that if a constant amplitude of modulation of stub length is used, the resulting amplitude of susceptance modulation will be a function of the average value of stub susceptance. For the present it will be assumed that the amplitude of susceptance modulation is independent of the average value of stub susceptance. This may actually be achieved by placing two stubs on the transmission line at the same position: One stub to provide the matching susceptance, the second to provide only the susceptance modulation.

With the two modulations defined, an example will be used to show that the modulation of position and susceptance each result in the generation of an amplitude modulation of the reflection coefficient, and that these components contain directional sense and become zero when the match pos-

ition and match value of susceptance are reached. The example will then be somewhat generalized to point out the properties of the two modulations. In general, changes in stub position and stub susceptance change the value of the total input admittance at the stub position, thus effecting a change in the reflection coefficient. The variations in total input admittance will be determined first, and then the effect of the changes in total input admittance on the reflection coefficient will be examined.

3. An example illustrating the use of the stub.

Consider the transmission line and shunt stub shown in Figure 10, where $\overline{K_L}$ is the complex load reflection coefficient and y_L is the load admittance in normalized units. The characteristic admittance of the transmission line is unity in normalized units.

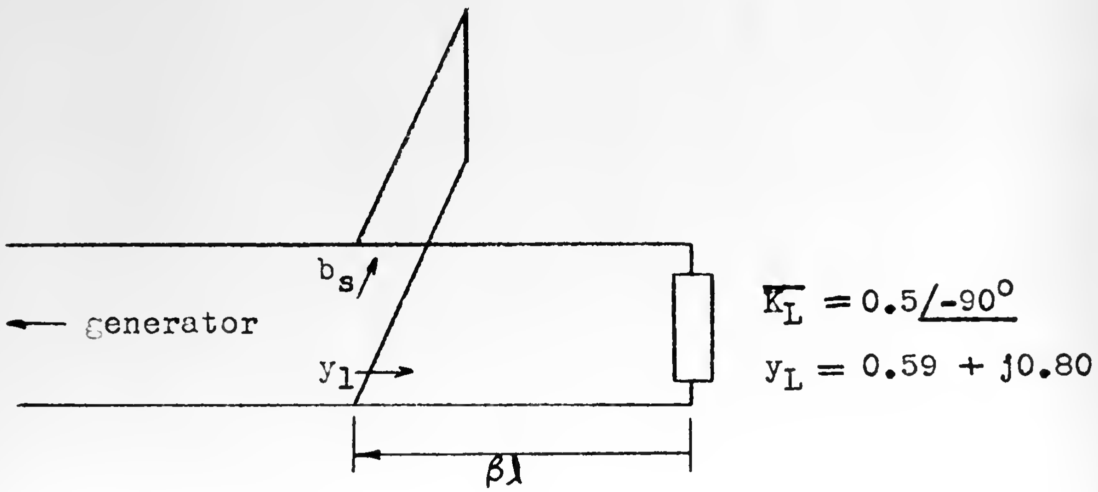


Figure 10

The transmission system used in the example.

From the Smith Chart, Figure 11, the load VSWR is read as 3.00, and the correct position for the stub for positive values of stub susceptance is computed

$$\beta l = (0.333 - 0.125)\lambda = 0.208\lambda \qquad 3\text{-}3$$

where λ is the wavelength along the transmission line, and depends on the frequency of operation and mode of propagation. The correct value of susceptance is read as 1.17 from the intersection of the K = 0.50 circle and the g_1 = 1.0 circle, where K is the magnitude of the reflection coefficient and g_1 is the conductance component of the input admittance of the transmission line at the stub position (not including the stub susceptance).

As has been described, there are two possible positions of match. Only one of these need be considered if the requirements of the system can be met by so doing. It will be shown later that the matching system can be adjusted to seek either match position, and that therefore only that match position corresponding to positive values of stub susceptance need be considered. Therefore, in what follows, it is assumed that only positive values of stub susceptance are required.

4. Position modulation; development of the position signal.

The effect of the position modulation is to cause the input admittance of the transmission line at the stub position (not including the stub susceptance) to vary along a

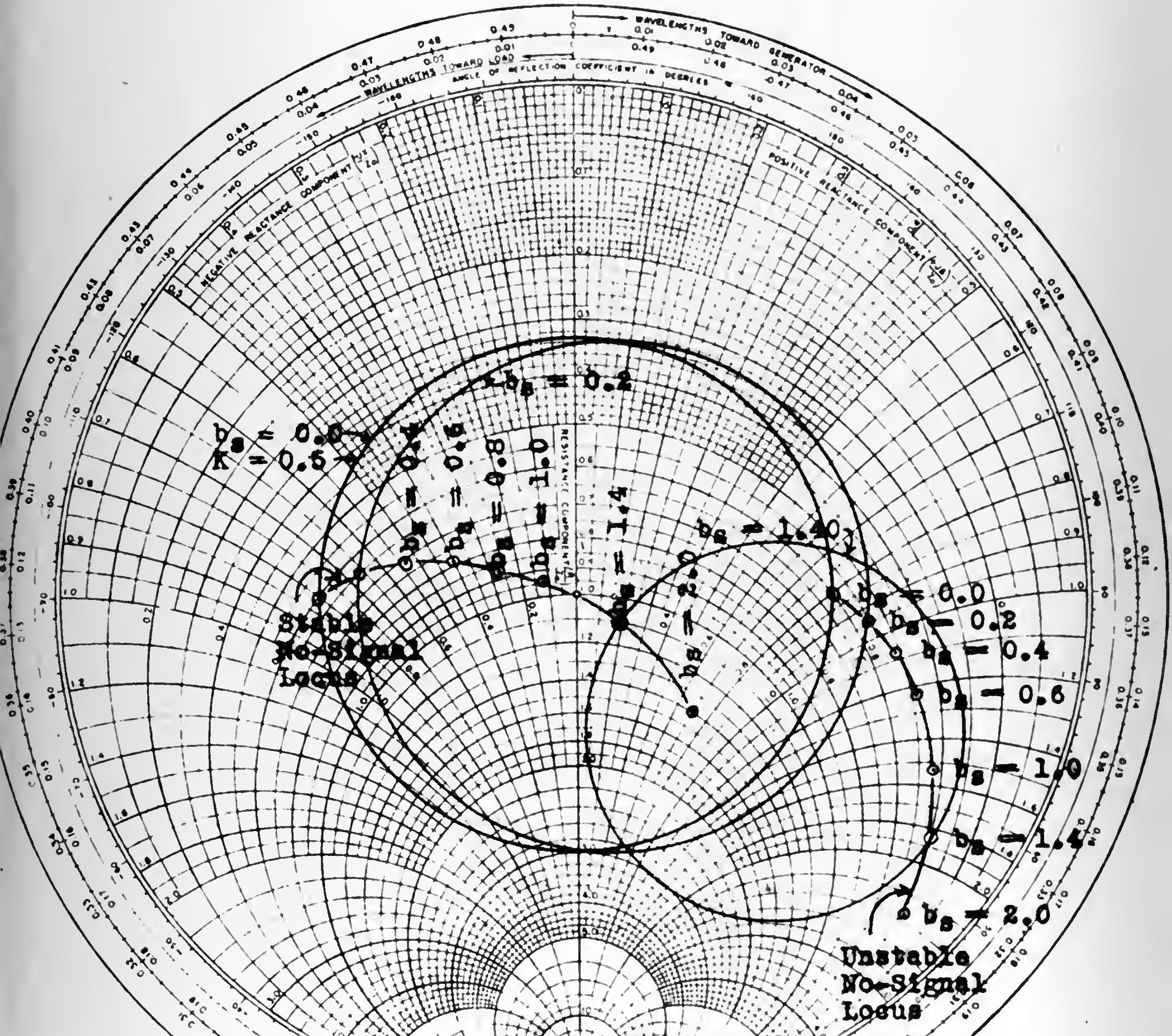


Figure 11

Smith Chart Presentation of the Example

short arc of the K = 0.50 circle about some point on the K = 0.50 circle corresponding to the value of $\overline{\beta \ell}$. Now the total input admittance at the stub position is the sum of the input admittance of the transmission line at the stub position (hereinafter called the line admittance) and the stub susceptance, and is given by

$$y_t = y_l + jb_s, \qquad 3\text{-}4$$

where y_t is the total input admittance at the stub position, and y_l is the line admittance and is the sum of the line conductance, g_l, and the line susceptance, b_l.

There are four cases to be considered:

(a) The stub susceptance is zero,

(b) The stub susceptance is greater than zero but less than the value required for match,

(c) The stub susceptance is equal to the value required for match, and

(d) The stub susceptance is greater than the value required for a match.

Consider first case (a) above, the stub susceptance is zero; then the position modulation causes the total input admittance to vary along a short arc of the circle of constant reflection coefficient magnitude, K = 0.50. Since this variation takes place along a circle of constant reflection coefficient magnitude it is apparent that when the stub susceptance is zero, position modulation does <u>not</u> result in ampli-

tude modulation of the reflection coefficient. This characteristic will make it necessary to provide a small value of stub susceptance at all times in order to ensure that the position modulation will at all times result in amplitude modulation of the reflection coefficient. This requirement will not have an adverse effect on the operation of the system and is easily met by providing the small continuous stub susceptance in the auxiliary modulation stub.

Next consider case (b), the stub susceptance is greater than zero, but less than the value required for match. As an example, assume that the stub susceptance is 0.20. The position modulation will cause the total input admittance to vary along a short arc of the curve labeled $b_s = 0.20$ in Figure 11 about some point on the curve corresponding to the value of $\overline{\beta \ell}$. Now a variation of the total input admittance along the curve $b_s = 0.20$ will result in amplitude modulation of the reflection coefficient at the modulation frequency except at those points at which the curve $b_s = 0.20$ is tangent to a circle of constant reflection coefficient magnitude. In Figure 11 there are two such points marked a and b. At either of these points, either an increases or a decrease in $\beta \ell$ will result in an increase in reflection coefficient; consequently the position modulation will result in amplitude modulation of the reflection coefficient at a fundamental frequency equal to twice the modulation frequency when $\beta \ell$ has the value corresponding to point a or to point b; the signi-

ficance of this fact is that there is no amplitude modulation of the reflection coefficient at the modulation frequency. For reasons which will be pointed out below, point a will be called a stable no-signal point, and point b will be called an unstable no-signal point. The corresponding positions of the stub will be called the stable no-signal position and the unstable no-signal position. If $\overline{\beta\ell}$ is greater than the value corresponding to point a, but less than the value corresponding to point b, an increase in $\beta\ell$ will result in an increase in the reflection coefficient, and a decrease in $\beta\ell$ will result in a decrease in the reflection coefficient; consequently the position modulation will result in amplitude modulation of the reflection coefficient at the modulation frequency and in phase with the position modulation. This amplitude modulation of the reflection coefficient in phase with the position modulation will be termed the positive phase position signal. If $\overline{\beta\ell}$ is less than the value corresponding to point a, but greater than the value corresponding to point b, an increase in $\beta\ell$ will result in a decrease in the reflection coefficient, and a decrease in $\beta\ell$ will result in an increase in the reflection coefficient; so that the position modulation results in amplitude modulation of the reflection coefficient at the modulation frequency but exactly 180 degrees out of phase with the position modulation. This amplitude modulation will be termed the negative phase position signal. The foregoing examination may be applied to each of

the curves of constant value of stub susceptance less than the match value; and in each case a point having the same characteristics as point a may be found. The locus of these points, termed the stable no-signal locus, is plotted in Figure 11, where it is seen that as the stub susceptance approaches the match value, this locus approaches the point $1 + j0$, the match value of the total input admittance. Thus, as the stub susceptance approaches the match value, the stable no-signal position of the stub, which results in amplitude modulation of the reflection coefficient at twice the modulation frequency, approaches the match position.

A similar locus of points having the same characteristics as point b may be determined, and this locus is also plotted in Figure 11. The significance of this locus will be explained below. This locus of the unstable no-signal points would also approach the match point if negative values of stub susceptance were permitted.

Now consider the third case above: The stub susceptance is equal to the value required for match. From what has gone before it is apparent that for this value of stub susceptance, point a is located at the point $1 + j0$. For positions of the stub other than the match position, the position modulation will result in the generation of the positive and negative phases of the position signal in exactly the same manner as above; that is, the positive phase position signal will be generated if $\overline{\beta \lambda}$ is greater than the match value, but less

than the value corresponding to point b, and the negative phase position signal will be generated if $\overline{\beta \ell}$ is less than the match value, but greater than the value corresponding to point b.

Lastly, consider the fourth case above: The stub susceptance is greater than the value required for a match. Assume that the stub susceptance is 1.40. Referring to Figure 11 once more, it is seen that the curve $b_s = 1.40$, along which the total input admittance varies as the stub position is modulated, has an external tangency to a curve of constant reflection coefficient magnitude at point a'. At this point the reflection coefficient will again be amplitude modulated at twice the modulation frequency. If the value of $\overline{\beta \ell}$ is greater than that corresponding to point a' (but less than the value corresponding to point b'), the positive phase position signal will be generated by the position modulation; if the value of $\overline{\beta \ell}$ is less than that corresponding to point a' (but greater than the value corresponding to point b'), the negative phase position signal will be generated by the position modulation; and therefore, point a' is also a stable no-signal position. This examination may be applied to each of the curves of constant value of stub susceptance greater than the match value; and in each case a point having the same characteristics as point a' may be found. The locus of these points is an extension of the locus of stable no-signal positions for values of stub susceptance less than the match

value. Again it is seen that as the stub susceptance approaches the match value, the stable no-signal positions approach the position of match.

It is now convenient to define the position signal more precisely: The position signal is the amplitude modulation of the reflection coefficient at the modulation frequency due to the position modulation of the stub. In view of this definition, and considering the examination of the effects of position modulation above, the following characteristics may be summarized:

(a) When $\overline{\beta l}$ is greater than the value corresponding to the stable no-signal position but less than the value corresponding to the unstable no-signal position, position modulation results in the generation of the positive phase position signal.

(b) When $\overline{\beta l}$ is less than the value corresponding to the stable no-signal position but greater than the value corresponding to the unstable no-signal position, position modulation results in the generation of the negative phase position signal.

(c) When $\overline{\beta l}$ has the value corresponding to either the stable or the unstable no-signal position, position modulation results in no position signal.

Thus it is apparent that the position modulation has resulted in the generation of a position signal which becomes zero when the stub is at the stable no-signal position (as

well as when the stub is at the unstable no-signal position), and which contains directional sense with respect to the stable no-signal position; it has also been shown that the stable no-signal position approaches the match position as the stub susceptance approaches the match value.

Now if the positive phase position signal is applied to the positioning servo loop in such a manner as to drive the stub toward the load, the negative phase position signal applied in the same manner will drive the stub toward the generator. The positive phase position signal is generated when the stub must be driven toward the load to reach the stable no-signal position; the negative phase position signal is generated when the stub must be driven toward the generator to reach the stable no-signal position. Therefore the application of either phase of the position signal will cause the stub to be driven toward the stable no-signal position. When the stub reaches the stable no-signal position, no position signal will be generated. If the stub is displaced from this position in either direction, a signal of the proper phase to return the stub to the stable no-signal position will be generated; thus the stub will seek and hold the stable no-signal position.

It is now necessary to consider the unstable no-signal positions. The position signal will be zero when the stub is at the position corresponding to either of the points <u>b</u> and <u>b'</u>. However, the locus of the points <u>b</u> does not approach the point

1 + j0 as the stub susceptance approaches the match value. Therefore, these points are ambiguous no-signal positions. It remains to be seen whether they are <u>stable</u> no-signal positions. Now if $\overline{\beta \lambda}$ is greater than the value corresponding to the point <u>b</u>, the position modulation will result in the generation of the <u>negative</u> phase position signal. But the negative phase position signal will drive the stub toward the generator, <u>away</u> from the position corresponding to the point <u>b</u>. If $\overline{\beta \lambda}$ is less than the value corresponding to the point <u>b</u>, the position modulation will result in the generation of the positive phase position which will again drive the stub away from the position corresponding to the point <u>b</u>. Thus the point <u>b</u> is seen to be an <u>un</u>stable no-signal position. If the stub should happen to be at the position corresponding to <u>b</u>, a slight displacement in either direction from that position would result in the correct phase of position signal to drive the stub to the stable no-signal position. An examination of the curves of constant stub susceptance for values of stub susceptance greater than the match value indicates that the points <u>b</u>' exhibit the same characteristics as are exhibited by the points <u>b</u>. The locus of unstable no-signal points is plotted in Figure 11.

It is now possible to summarize the effects of position modulation since these effects have been examined for various values of stub susceptance and for various stub positions;

(a) For each value of stub susceptance there is a sta-

ble no-signal position which the stub will seek and hold if the position signals are properly applied in a servo loop controlling the position of the stub.

(b) For each value of stub susceptance there is an unstable no-signal position, but a slight displacement of the stub from this position in either direction will result in the generation of a position signal of the proper phase to drive the stub to the stable no-signal position if the position signals are properly applied.

(c) The stable no-signal position which the stub will seek and hold is not the match position unless the stub susceptance has the match value; but the stable no-signal position approaches the match position as the stub susceptance approaches the match value.

In order to generalize the results of this qualitative analysis, the loci of stable and unstable no-signal positions for several values of load reflection coefficient magnitude are plotted in Figure 12.

The examinations of this section were based on the assumption that the stub was limited to positive values of susceptance. A similar analysis may be carried out based on the assumption that the stub is limited to negative values of susceptance; such an analysis would reveal the same characteristics as are outlined above. The match position would not, of course, be the same as for positive values of stub susceptance.

5. Susceptance modulation; development of the susceptance sig-

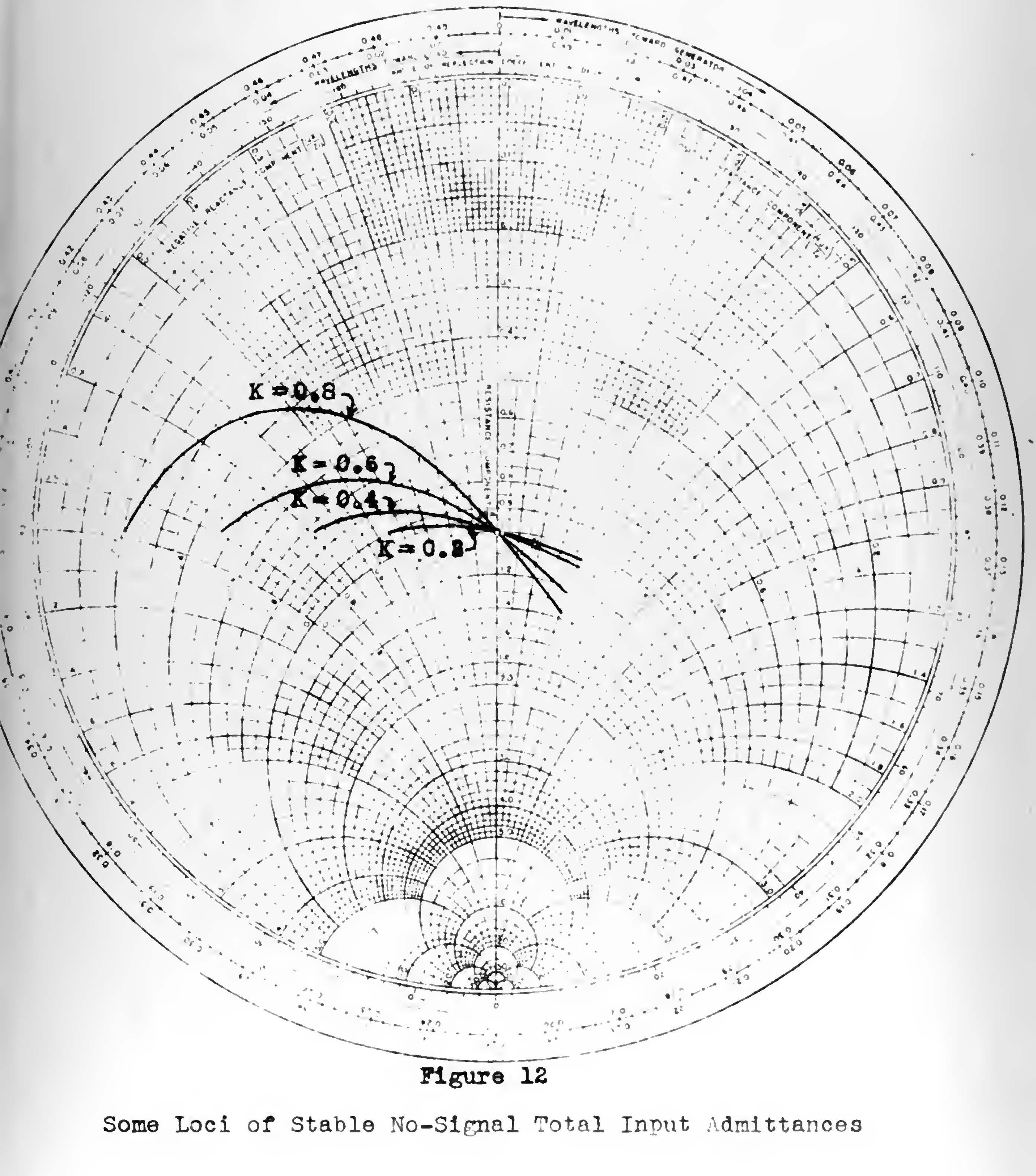


Figure 12

Some Loci of Stable No-Signal Total Input Admittances

nal.

The effect of the susceptance modulation is to cause the total input admittance to vary along a short arc of the circle of constant conductance equal to the line conductance, about a point corresponding to the value of $\overline{b_s}$. This is apparent from equation 3-4

$$y_t = y_l + jb_s, \qquad 3\text{-}4$$

where the instantaneous stub susceptance is given by equation 3-2:

$$b_s = \overline{b_s} + \Delta b\cos\omega_m t. \qquad 3\text{-}2$$

There are four cases to be considered:

(a) The stub position is such that the line susceptance is negative,

(b) The stub position is such that the stub is at a voltage maximum of the standing wave on the transmission line,

(c) The stub position is such that the line susceptance is positive, and

(d) The stub position is such that the stub is at a voltage minimum of the standing wave on the transmission line.

Consider first case (a) above, the stub position is such that the line susceptance is negative; then the susceptance modulation causes the total input admittance to vary along a short arc of a circle of constant conductance equal to the

[illegible]

line conductance <u>in the left half of the admittance plane</u>, when the value of $\overline{b_s}$ is less than the magnitude of the line susceptance. If the stub susceptance is increased, the reflection coefficient is decreased; if the stub susceptance is decreased, the reflection coefficient is increased; thus the susceptance modulation will result in an amplitude modulation of the reflection coefficient at the modulation frequency but exactly 180 degrees out of phase with the susceptance modulation. This will be termed the negative phase susceptance signal.

Now along the degenerate circle of zero susceptance on the Smith Chart, the circles of constant conductance are tangent to the circles of constant reflection coefficient magnitude. If the stub susceptance is equal to the magnitude of the line susceptance, the total input susceptance is zero; then either an increase or a decrease in stub susceptance will result in an increase in the reflection coefficient. The amplitude modulation of the reflection coefficient thus generated by the susceptance modulation will have a fundamental frequency twice the modulation frequency; this is true regardless of the value of the line conductance. Thus the degenerate circle of zero susceptance constitutes a locus of values of total input admittance at which susceptance modulation will result in no amplitude modulation of the reflection coefficient at the modulation frequency. The value of $\overline{b_s}$ which yields this condition will be termed the stable no-

signal value; the fact that this is a stable no-signal value will be shown below.

Now consider that the stub susceptance is greater than the magnitude of the line susceptance. The total input admittance is then in the right half of the admittance plane. If the stub susceptance is increased, the reflection coefficient will be increased; if the stub susceptance is decreased, the reflection coefficient will be decreased. Thus the susceptance modulation will result in an amplitude modulation of the reflection coefficient at the modulation frequency and in phase with the susceptance modulation. This will be termed the positive phase susceptance signal.

Now if the positive phase susceptance signal is applied to the susceptance servo loop in such a manner as to decrease the value of $\overline{b_s}$, the negative phase susceptance signal applied in the same manner will increase the value of $\overline{b_s}$. The positive phase susceptance signal is generated when the value of $\overline{b_s}$ is greater than the stable no-signal value; the negative phase susceptance signal is generated when the value of $\overline{b_s}$ is less than the stable no-signal value. Therefore, the application of either phase of the susceptance signal will cause the value of $\overline{b_s}$ to change toward the stable no-signal value. When the value of $\overline{b_s}$ reaches the stable no-signal value, no susceptance signal will be generated. If the value of $\overline{b_s}$ increases or decreases, a signal of the proper phase to return the value of $\overline{b_s}$ to the stable no-signal value will be

generated; thus the value of $\overline{b_s}$ will seek and hold the stable no-signal value.

Now consider the second case above: The stub position is such that the stub is at a voltage maximum of the standing wave on the transmission line. In this case the line susceptance is zero. If the stub susceptance is greater than zero, the total input admittance will be in the right half of the admittance plane, and susceptance modulation will result in the generation of the positive phase susceptance signal. This signal, applied properly to the servo loop, will reduce the value of $\overline{b_s}$ until the stable no-signal value is reached; in this case until $\overline{b_s}$ is zero. When $\overline{b_s}$ reaches zero the signal becomes zero, and any change in the value of $\overline{b_s}$ will result in the generation of a susceptance signal of the proper phase to return $\overline{b_s}$ to zero.

Next consider the third case above: The stub position is such that the line susceptance is positive. Since it has been assumed that the stub susceptance is limited to positive values, the total input admittance will always be in the right half of the admittance plane. If the stub susceptance is increased, the reflection coefficient will be increased; if the stub susceptance is decreased, the reflection coefficient will be decreased. Thus, regardless of the magnitude of the line susceptance or stub susceptance, the positive phase susceptance signal will always be generated by the susceptance modulation. This signal will cause the value of $\overline{b_s}$ to be re-

duced. When the value of $\overline{b_s}$ reaches zero it will have attained its minimum value; the stub susceptance cannot become negative. But the total input admittance will still have a positive component of susceptance, so that the positive phase susceptance signal will continue to be generated. This characteristic will require that some means be provided to interrupt the signal when the value of $\overline{b_s}$ reaches zero, even though the match condition has not been reached. The stub position, in this case, is such that the total input admittance is in the wrong half of the admittance plane to achieve a match. But it has been shown in the discussion of position modulation above that when the stub is in this adverse region, the correct phase of the position signal to drive the stub into the favorable region will be generated. When the stub reaches a position such that the total input admittance is in the left half plane; that is, the line susceptance is negative, the correct susceptance signal will be generated to drive the stub susceptance to the stable no-signal value. The difficulty described in this paragraph is a result of limiting the stub to positive values of susceptance.

Finally, consider the fourth case above: The stub position is such that the stub is located at a voltage minimum of the standing wave on the transmission line; that is, the line susceptance is zero and the line conductance is greater than one. This case is identical to the second case, where the line susceptance was zero but the line conductance was

less than one. In this case, if the value of $\overline{b_s}$ is greater than zero, the positive phase susceptance signal will be generated and the stub susceptance will be reduced to zero, the stable no-signal value.

It is now possible to summarize the effects of the susceptance modulation since these effects have been examined for various stub positions and for various values of $\overline{b_s}$. It should be noted that the analysis contained in this section has not been restricted to the example of the previous two sections, but has been general.

(a) When the stub position is such that the line susceptance is zero or negative, the correct phase of susceptance to drive the stub susceptance to the stable no-signal value will be generated, regardless of the exact position of the stub.

(b) When the stub position is such that the line susceptance is positive, the susceptance signal generated will drive the stub susceptance to zero initially. When the stub has been driven into the region such that the line susceptance is negative, the stub susceptance will be driven to the stable no-signal value.

(c) The amplitude of the susceptance signal is not proportional to the value of stub susceptance as the position signal was, although the amplitude is related to the stub susceptance in a minor way.

(d) The amplitude of the susceptance signal is a func-

tion of the value of line conductance, and thus is a function of the stub position.

6. Position and susceptance signals - conclusion.

The qualitative analysis of the previous sections has considered the effects of the position and susceptance modulations independently. It is apparent from what has been shown of the characteristics of the modulations that they are not independent, and that some investigation of their interdependence should be made. This investigation will be made at the conclusion of the mathematical analysis of the next chapter.

Furthermore, nothing has been said about the amplitudes of the two modulations, although they have been assumed to be small. The limiting amplitudes for proper operation will be determined in the next section on the basis of the Smith Chart presentation, and in Chapter IV on the basis of the mathematical analysis of that chapter. The maximum required value of stub susceptance will also be determined in the next section after setting some design values.

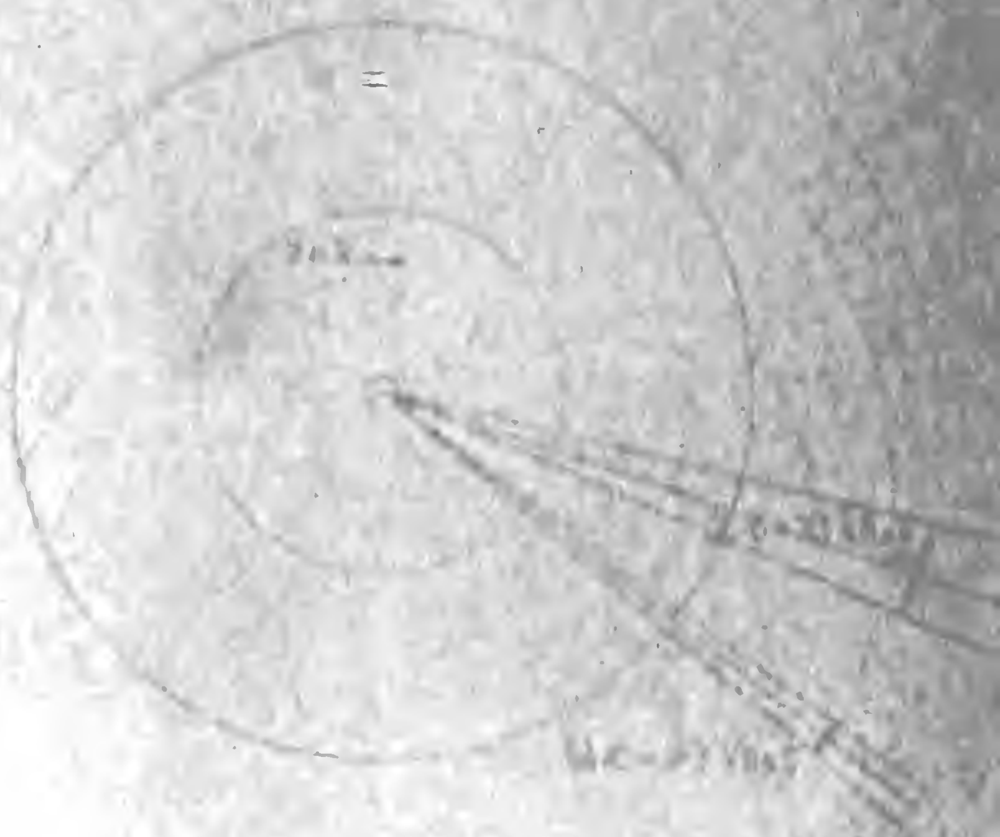
7. Design criteria and resulting values of parameters.

In order to determine the amplitudes of the two modulations, and the maximum required value of stub susceptance, it is necessary to establish some design criteria. The desirable characteristics of the automatic matching system have been outlined in section 3 of Chapter I. The first of these was:

(a) Provide a very low VSWR even when the load reflection coefficient is moderately high, say 0.60, corresponding to a load VSWR of 4.0.

It will be assumed that the maximum load VSWR which the automatic system will be required to match is 4.0 ($K_L = 0.60$). It will further be assumed that the system is required to have a maximum VSWR presented to the generator of 1.05 when the stub position and susceptance have reached the match position and match value respectively. This value of VSWR corresponds approximately to a magnitude of reflection coefficient of 0.025.

At the matched condition, the average value of the reflection coefficient is determined by the amplitudes of the two modulations. The effect of the two modulations, since they are 90 degrees out of phase, will be to cause the total input admittance to follow a closed path in the admittance plane. At the matched condition, this path will be very nearly circular if the modulations have appropriately chosen amplitudes; the center of the path will be the point $1 + j0$ on the admittance plane. If the maximum allowable value of the reflection coefficient at the matched condition is 0.025, then this closed path must be contained within the circle of constant reflection coefficient magnitude equal to 0.025. This circle of constant reflection coefficient magnitude is shown in the Smith Chart of Figure 13, and the construction of Figure 13 shows the determination of the amplitudes of the two



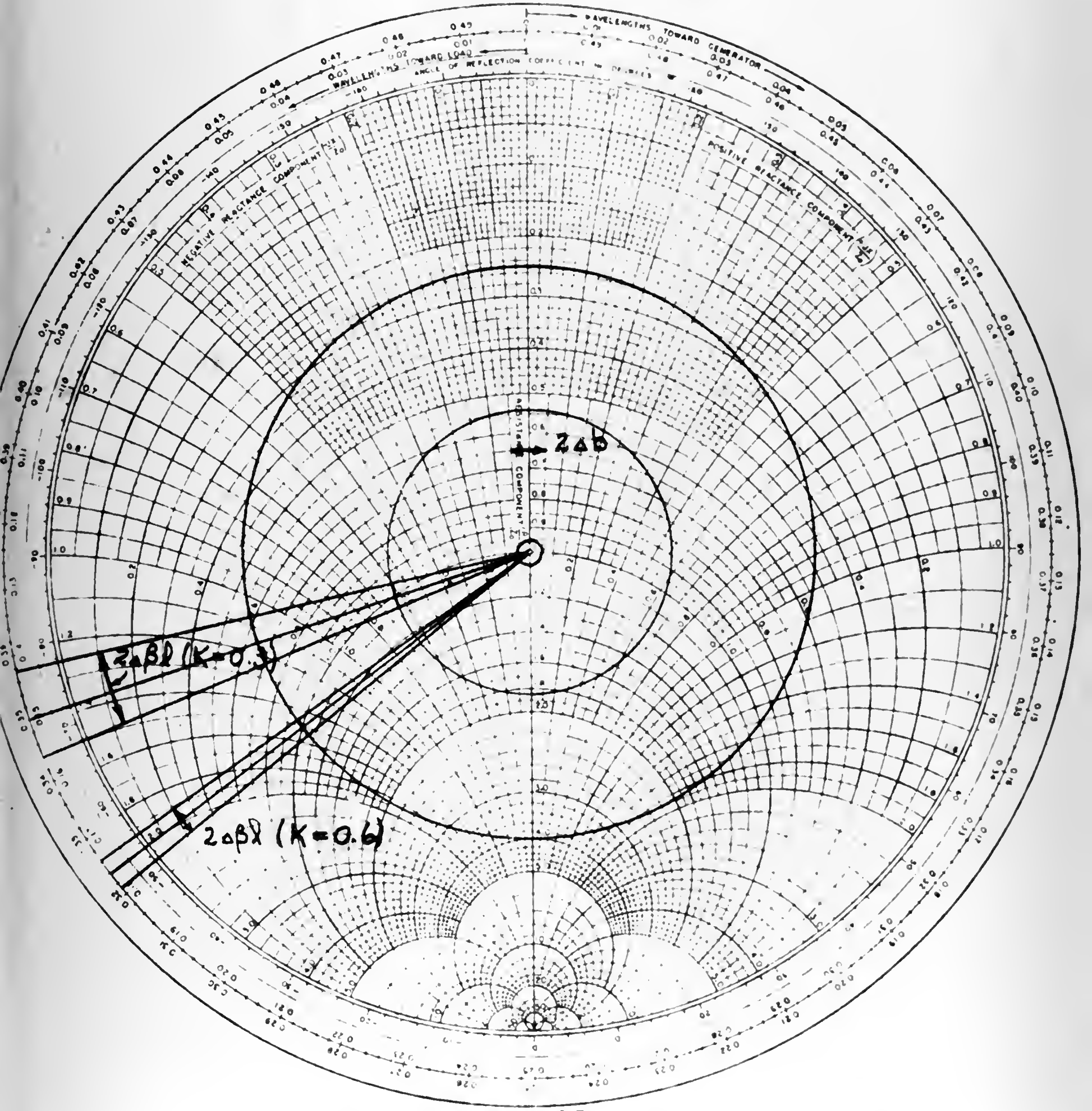


Figure 13

The Determination of the Modulation Amplitudes

modulations for a load reflection coefficient magnitude of 0.60, the design maximum. These amplitudes are:

$$\Delta\beta\ell = 0.003\lambda, \text{ and} \tag{3-5}$$

$$\Delta b = 0.05. \tag{3-6}$$

Figure 13 also shows that the amplitude modulation of the reflection coefficient due to the position modulation is proportional to the magnitude of the load reflection coefficient. To illustrate this point, the required position modulation amplitude to yield an average reflection coefficient of 0.025 when the load reflection coefficient magnitude is 0.30 is determined to be 0.007λ. The construction is shown in Figure 13.

Also from Figure 13, the maximum required value of stub susceptance to match a load VSWR of 4.0 may be determined from the magnitude of the line susceptance when the stub is at the match position. This maximum required value of stub susceptance is 1.50 normalized units.

8. Conclusion.

The characteristics of the two modulations have been determined in a qualitative manner and are summarized on pages 43, 44, 51, and 52.

It should be noted here that the amplitude of the reflected wave is also directly proportional to the amplitude of the incident wave. Thus the amplitude modulation of the reflected wave is directly proportional to the amplitude of the incident wave and the amplitude modulation of the re-

flection coefficient. Therefore, as the amplitude of the incident wave is increased, the amplitude modulation of the reflection coefficient may be decreased proportionately. Thus the required modulation amplitudes are inversely proportional to the amplitude of the incident wave; so that in a high power system the VSWR at the matched condition may be so small as to be unmeasurable even though the modulation amplitudes are large enough to provide adequate signals. Conversely, in a low power system the VSWR at the matched condition may be larger than the value, 1.05, specified above, when the modulation amplitudes are adjusted to provide adequate signals. These characteristics result from the fact that approximately equal signal amplitudes will be required regardless of the amplitude of the incident wave.

The characteristics of the signals generated by the two modulations are briefly summarized below:

Position signals: For each magnitude of load reflection coefficient there is a locus of stable no-signal positions of the stub. Each of these stable positions corresponds to a value of stub susceptance. As the stub susceptance approaches the match value, the stable no-signal position approaches the match position. For each magnitude of load reflection coefficient there is a locus of ambiguous, unstable, no-signal positions. Each of these unstable positions corresponds to a value of stub susceptance. A slight displacement of the stub from one of these unstable positions will result in the

[illegible]

generation of the correct signal to drive the stub toward the match position. The amplitude of the positioning signal is proportional to the value of the stub susceptance and is inversely proportional to the magnitude of the load reflection coefficient.

Susceptance signals: The locus of stable no-signal values of total input admittance is the circle of zero total input susceptance. There are no unstable no-signal values of stub susceptance. The amplitude of the susceptance signal is related to the position of the stub, and is a maximum when the stub is at such a position that the phase angle of the total reflection coefficient is approximately 90 degrees. This corresponds fairly well with the loci of stable no-signal values of total input admittance for position modulation, so that when the stub reaches the stable no-signal position the susceptance signal will have approximately its maximum amplitude.

General: It is to be noted that the position which the stub seeks is not the position of match unless the stub susceptance has the match value; and that the value of susceptance which the stub seeks is not the match value unless the stub is at the match position. This is an indication of the interdependence of stub susceptance and stub position which was mentioned in Chapter II in the discussion of the manual adjustment of the single stub tuner by observation of the magnitude of the reflected wave.

CHAPTER IV

ANALYSIS OF THE POSITION AND SUSCEPTANCE MODULATIONS

1. Introduction.

Before proceeding with the analysis of the modulations, an analysis of the single stub tuner will be made to determine analytically the stub susceptance and position for a match in terms of the complex reflection coefficient of the load. Normalized admittances will be used throughout these analyses.

The single stub tuner and associated transmission line may be represented schematically as shown in Figure 14. For ease of analysis, the origin, a, is taken at the position of a voltage minimum, since at this point the reflection coefficient is real, that is,

$$\overline{K_a} = -K_L. \qquad 4\text{-}1$$

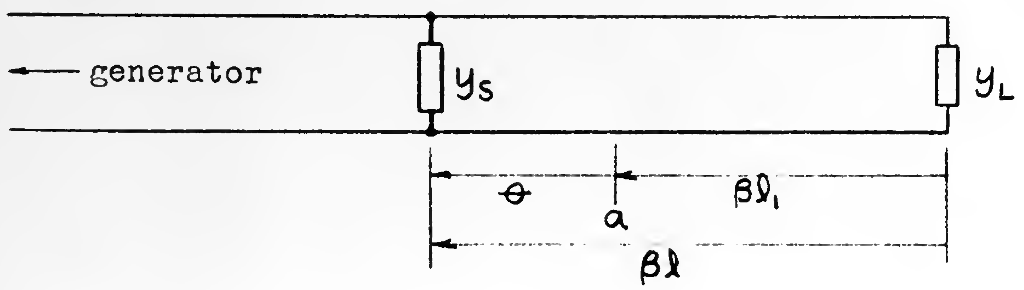


Figure 14

Schematic representation of the single stub tuner and associated transmission line.

The complex load reflection coefficient is given in

terms of the load admittance by

$$\bar{K}_L = \frac{1 - y_L}{1 + y_L} \tag{4-2}$$

and the load admittance is given in terms of the complex load reflection coefficient by

$$y_L = \frac{1 - \bar{K}_L}{1 + \bar{K}_L} . \tag{4-3}$$

At the origin, the transformed complex load reflection coefficient is

$$\bar{K}_a = \bar{K}_L \epsilon^{-j2\beta l_1} = K_L \epsilon^{j\varphi_L} \epsilon^{-j2\beta l_1} = K_L \epsilon^{-j\pi} , \tag{4-4}$$

so that the electrical distance from the load to the position of a voltage minimum is given by

$$\beta l_1 = \frac{1}{2}(\pi + \varphi_L) . \tag{4-5}$$

Any integral number of half wavelengths may be added to this value of βl_1 since the voltage standing wave has minima every half wavelength along the transmission line. In equation 4-5, φ_L is the phase angle of the complex load reflection coefficient. It is apparent that the distance βl_1 is determined only by the load admittance, y_L.

The analysis of the single stub tuner will include the determination of the load to stub separation, βl, and the stub susceptance, b_s, in terms of the load reflection coefficient; and determination of the variation of the reflection coefficient at the stub position due to the modulations

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imposed on the system. In making this determination, it will be assumed that the two modulations are independent, and they will be considered separately. After determining the effects of the modulations, the cross-talk between the two signals will be examined.

2. The conditions for a match at the stub position.

The admittance of the line at the origin is given by

$$y_a = \frac{1 - \overline{K_a}}{1 + \overline{K_a}} . \qquad 4\text{-}6$$

Since $\overline{K_{La}} = -K_L$, this becomes

$$y_a = \frac{1 + K_L}{1 - K_L} = g_a + j\,0 . \qquad 4\text{-}7$$

Transforming this admittance to an arbitrary position of the stub yields

$$y_\theta = \frac{y_a + j\tan\theta}{1 + j\,y_a \tan\theta} \qquad 4\text{-}8$$

$$= \frac{(1+K_L)\cos\theta + j(1-K_L)\sin\theta}{(1-K_L)\cos\theta + j(1+K_L)\sin\theta} . \qquad 4\text{-}9$$

Rationalizing,

$$y_\theta = \frac{1 - K_L^2}{1 + K_L^2 - 2K_L\cos 2\theta} - j\,\frac{2K_L\sin 2\theta}{1 + K_L^2 - 2K_L\cos 2\theta} . \qquad 4\text{-}10$$

It is assumed that the stub is also lossless, so that the stub admittance is given by

$$y_s = 0 + j\,b_s . \qquad 4\text{-}11$$

Then the total input admittance at the arbitrary stub position becomes

$$y_t = \frac{1 - K_L^2}{1 + K_L^2 - 2K_L \cos 2\theta} + j\left[b_s - \frac{2K_L \sin 2\theta}{1 + K_L^2 - 2K_L \cos 2\theta}\right]. \qquad 4\text{-}12$$

The conditions for a match at the stub position are:

$$\text{(a)} \quad g_t = 1, \qquad 4\text{-}13$$

$$\text{(b)} \quad b_t = 0. \qquad 4\text{-}14$$

Then from equations 4-12 and 4-13

$$\frac{1 - K_L^2}{1 + K_L^2 - 2K_L \cos 2\theta} = 1, \qquad 4\text{-}15$$

which yields

$$\theta = \pm \frac{\arccos K_L}{2}. \qquad 4\text{-}16$$

From equations 4-12 and 4-14,

$$b_s = \frac{2K_L \sin 2\theta}{1 + K_L^2 - 2K_L \cos 2\theta}. \qquad 4\text{-}17$$

Substituting from equation 4-16 and using the relationships

$$\cos 2\theta = K_L$$

$$\sin 2\theta = \pm\sqrt{1 - K_L^2}$$

equation 4-17 yields

$$b_s = \pm \frac{2K_L}{\sqrt{1 - K_L^2}}. \qquad 4\text{-}18$$

Equation 4-16 has two solutions for each magnitude of K_L, corresponding to the two intersections of the circle of unity

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conductance on the admittance plane with the circle of constant K equal to K_L. One of these intersections is in the positive susceptance half plane and the other in the negative susceptance half plane. Since it has been assumed that the stub susceptance can take only positive values the solution of interest is that associated with the negative susceptance half plane and therefore the positive solution of equation 4-16 is the desired value of θ. This gives positive values of stub susceptance from equation 4-18.

The conditional values of stub position, βl, measured from the load toward the generator, and stub susceptance have now been determined. They are:

$$\beta l = \frac{1}{2}\left[\arccos K_L + \pi + \varphi_L\right] \qquad 4\text{-}19$$

$$b_s = \frac{2K_L}{\sqrt{1-K_L^2}} \, . \qquad 4\text{-}20$$

3. The reflection coefficient at the stub position.

It will be assumed that a square law detector will be used in the automatic matching system. Thus the detector output, which constitutes the signal for the servo amplifier, will be proportional to the square of the magnitude of the reflected wave, which is in turn proportional to the square of the magnitude of the reflection coefficient at the stub position. Therefore, this quantity will be determined prior to investigating the effects of the modulations.

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The admittance at the stub position is given by equation 4-12

$$y_t = \frac{1 - K_L^2}{1 + K_L^2 - 2K_L \cos 2\theta} + j\left[b_s - \frac{2K_L \sin 2\theta}{1 + K_L^2 - 2K_L \cos 2\theta}\right]. \qquad 4\text{-}12$$

Then the reflection coefficient at the stub position is

$$\bar{K}_t = \frac{1 - \dfrac{1 - K_L^2}{1 + K_L^2 - 2K_L \cos 2\theta} - j\left[b_s - \dfrac{2K_L \sin 2\theta}{1 + K_L^2 - 2K_L \cos 2\theta}\right]}{1 + \dfrac{1 - K_L^2}{1 + K_L^2 - 2K_L \cos 2\theta} + j\left[b_s - \dfrac{2K_L \sin 2\theta}{1 + K_L^2 - 2K_L \cos 2\theta}\right]} \qquad 4\text{-}21$$

$$= \frac{2K_L^2 - 2K_L \cos 2\theta - j\left[b_s(1 + K_L^2 - 2K_L \sin 2\theta) - 2K_L \sin 2\theta\right]}{2 - 2K_L \cos 2\theta + j\left[b_s(1 + K_L^2 - 2K_L \sin 2\theta) - 2K_L \sin 2\theta\right]} \qquad 4\text{-}22$$

$$= \frac{\bar{N}}{\bar{D}} . \qquad 4\text{-}23$$

And

$$K_t^2 = \frac{N^2}{D^2} \qquad 4\text{-}24$$

where

$$N^2 = 4K_L^2(K_L^2 - 2K_L \cos 2\theta + \cos^2 2\theta) + b_s(1 + K_L^2 - 2K_L \cos 2\theta)^2 - 4K_L b_s \sin 2\theta (1 + K_L^2 - 2K_L \cos 2\theta) + 4K_L^2 \sin^2 2\theta \qquad 4\text{-}25$$

$$= 4K_L^2(1 + K_L^2 - 2K_L \cos 2\theta) + b_s^2(1 + K_L^2 - 2K_L \cos 2\theta)^2 - 4K_L b_s \sin 2\theta (1 + K_L^2 - 2K_L \cos 2\theta) \qquad 4\text{-}26$$

and

$$D^2 = 4(1 - 2K_L \cos 2\theta + K_L^2 \cos^2 2\theta) + b_s^2(1 + K_L^2 - 2K_L \cos 2\theta)^2 - 4K_L b_s \sin 2\theta (1 + K_L^2 - 2K_L \cos 2\theta) + 4K_L^2 \sin^2 2\theta \qquad 4\text{-}27$$

$$D^2 = 4(1 + K_L^2 - 2K_L\cos 2\theta) + b_s^2(1+K_L^2 - 2K_L\cos 2\theta)^2$$
$$- 4K_L b_s \sin 2\theta(1+K_L^2 - 2K_L\cos 2\theta) . \qquad 4\text{-}28$$

Therefore,

$$K_t^2 = \frac{4K_L^2 + b_s^2(1+K_L^2 - 2K_L\cos 2\theta) - 4K_L b_s \sin 2\theta}{4 + b_s^2(1+K_L^2 - 2K_L\cos 2\theta) - 4K_L b_s \sin 2\theta} . \qquad 4\text{-}29$$

Two components of modulation are to be introduced into this expression. These are

(a) $b_s = \overline{b_s} + \Delta b \cos\omega_m t$ 3-2

and (b) $\beta\ell = \overline{\beta\ell} + \Delta\beta\ell \sin\omega_m t$. 3-1

But since $\beta\ell$ is equal to θ to within an additive constant, the position modulation may be taken as imposed on the electrical distance θ, so that

$$\theta = \overline{\theta} + \Delta\theta \sin\omega_m t \qquad 4\text{-}30$$

where θ is the instantaneous electrical distance from a voltage minimum to the stub position, $\overline{\theta}$ is the average distance from the same voltage minimum to the stub position, and $\Delta\theta$ is the amplitude of position modulation.

4. Analysis of the susceptance modulation.

In order to analyze the effects of the stub susceptance modulation given by equation 3-2, it is necessary to substi-

tute equation 3-2 into equation 4-29. Rewriting equation 4-29 in slightly different form,

$$K_t^2 = \frac{4K_L^2 - 4K_L b_s \sin 2\theta + b_s^2(1+K_L^2 - 2K_L\cos 2\theta)}{4 - 4K_L b_s \sin 2\theta + b_s^2(1+K_L^2-2K_L\cos 2\theta)} \qquad 4\text{-}31$$

$$= \frac{N'}{D'} . \qquad 4\text{-}32$$

Substituting equation 3-2 and evaluating the numerator of the resulting expression,

$$N' = 4K_L^2 - 4K_L(\bar{b}_s + \Delta b\cos\omega_m t)\sin 2\theta$$

$$+ (\bar{b}_s + \Delta b\cos\omega_m t)^2(1+K_L^2 - 2K_L\cos 2\theta) \qquad 4\text{-}33$$

$$= 4K_L^2 - 4K_L(\bar{b}_s + \Delta b\cos\omega_m t)\sin 2\theta$$

$$+\left[b_s^2 + 2\bar{b}_s\Delta b\cos\omega_m t + (\Delta b)^2\cos^2\omega_m t\right](1+K_L^2-2K_L\cos 2\theta) \qquad 4\text{-}34$$

$$= 4K_L^2 - 4K_L\bar{b}_s\sin 2\theta + \bar{b}_s^{\,2}(1+K_L^2-2K_L\cos 2\theta)$$

$$+\Delta b\cos\omega_m t\left[2\bar{b}_s(1+K_L^2-2K_L\cos 2\theta) - 4K_L\sin 2\theta\right]$$

$$+(\Delta b)^2\cos^2\omega_m t\,(1+K_L^2-2K_L\cos 2\theta) \qquad 4\text{-}35$$

$$= 4K_L^2 - 4K_L\bar{b}_s\sin 2\theta + \left(\bar{b}_s^{\,2} + \frac{(\Delta b)^2}{2}\right)(1+K_L^2-2K_L\cos 2\theta)$$

$$+\Delta b\left[2\bar{b}_s(1+K_L^2-2K_L\cos 2\theta) - 4K_L\sin 2\theta\right]\cos\omega_m t$$

$$+\frac{(\Delta b)^2}{2}(1+K_L^2-2K_L\cos 2\theta)\cos 2\omega_m t \qquad 4\text{-}36$$

or

$$N' = 4K_L^2 + N_0' + N_1' \cos\omega_m t + N_2' \cos 2\omega_m t \; ; \qquad 4\text{-}37$$

and similarly for the denominator of equation 4-31,

$$D' = 4 - 4K_L \bar{b}_s \sin 2\theta + \left(\bar{b}_s + \frac{(\Delta b)^2}{2}\right)\left(1 + K_L^2 - 2K_L \cos 2\theta\right)$$
$$+ \Delta b\left[2\bar{b}_s(1+K_L^2 - 2K_L\cos 2\theta) - 4K_L \sin 2\theta\right]\cos\omega_m t$$
$$+ \frac{(\Delta b)^2}{2}\left[1 + K_L^2 - 2K_L\cos 2\theta\right]\cos 2\omega_m t \qquad 4\text{-}38$$

$$= 4 + N_0' + N_1' \cos\omega_m t + N_2' \cos 2\omega_m t. \qquad 4\text{-}39$$

So that

$$K_t^2 = \frac{4K_L^2 + N_0' + N_1'\cos\omega_m t + N_2'\cos 2\omega_m t}{4 + N_0' + N_1'\cos\omega_m t + N_2'\cos 2\omega_m t} \qquad 4\text{-}40$$

where

$$N_0' = -4K_L\bar{b}_s \sin 2\theta + \left(\bar{b}_s + \frac{(\Delta b)^2}{2}\right)\left(1 + K_L^2 - 2K_L\cos 2\theta\right), \qquad 4\text{-}41$$

$$N_1' = \Delta b\left[2\bar{b}_s(1+K_L^2 - 2K_L\cos 2\theta) - 4K_L\sin 2\theta\right] \qquad 4\text{-}42$$

$$N_2' = \frac{(\Delta b)^2}{2}\left[1 + K_L^2 - 2K_L\cos 2\theta\right] \qquad 4\text{-}43$$

First examine equation 4-40 at the match conditions. To do this substitute equations 4-19 and 4-20 in equations 4-41 through 4-43, obtaining for N_0'

$$N_0' = -4K_L \frac{2K_L}{\sqrt{1-K_L^2}}\sqrt{1-K_L^2} + (1-K_L^2)\frac{4K_L^2}{1-K_L^2} + \frac{(\Delta b)^2}{2}(1-K_L^2) \qquad 4\text{-}44$$

$$= -4K_L^2 + \frac{(\Delta b)^2}{2}(1-K_L^2) \qquad 4\text{-}45$$

and for N_1'

$$N_1' = \Delta b\left[2\frac{2K_L}{\sqrt{1-K_L^2}}(1-K_L^2) - 4K_L\sqrt{1-K_L^2}\right] = 0 \qquad 4\text{-}46$$

and for N_2'

$$N_2' = \frac{(\Delta b)^2}{2}(1-K_L^2)\,. \qquad 4\text{-}47$$

So that

$$K_{t_M}^2 = \frac{4K_L^2 - 4K_L^2 + \frac{(\Delta b)^2}{2}(1-K_L^2) + \frac{(\Delta b)^2}{2}(1-K_L^2)\cos 2\omega_m t}{4 - 4K_L^2 + \frac{(\Delta b)^2}{2}(1-K_L^2) + \frac{(\Delta b)^2}{2}(1-K_L^2)\cos 2\omega_m t} \qquad 4\text{-}48$$

$$= \frac{\frac{(\Delta b)^2}{2}(1-K_L^2)(1+\cos 2\omega_m t)}{(1-K_L^2)\left(4 + \frac{(\Delta b)^2}{2} + \frac{(\Delta b)^2}{2}\cos 2\omega_m t\right)} \qquad 4\text{-}49$$

$$= \frac{\frac{(\Delta b)^2}{2}(1+\cos 2\omega_m t)}{4 + \frac{(\Delta b)^2}{2}(1+\cos 2\omega_m t)}\,. \qquad 4\text{-}50$$

From this expression it is apparent that at the match conditions there is no signal component at the fundamental modulation frequency. The average value of $K_{t_M}^2$ is very small, approximately $\frac{(\Delta b)^2}{8}$. There is a signal at double the modulation frequency which also has an amplitude of approximately $\frac{(\Delta b)^2}{8}$. It should be noted that the average value of $K_{t_M}^2$ is in-

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dependent of the variable parameters of the system; θ, $\overline{b_s}$, and K_L.

From equation 4-50 the maximum value of Δb, based on the maximum permissible peak value of K_t^2, can be determined. It is assumed that the peak value of K_t^2 is to be that value corresponding to a VSWR of 1.05.

Then

$$K_{t_M} = \frac{1.05 - 1}{1.05 + 1} = \frac{0.05}{2.05} \doteq 0.025 \qquad 4\text{-}51$$

$$K_{t_M}^2 = (0.025)^2 = \frac{\frac{(\Delta b)^2}{2}(1+1)}{4 + \frac{(\Delta b)^2}{2}(1+1)} . \qquad 4\text{-}52$$

If Δb is small compared to one,

$$(\Delta b)^2 = 4\,(0.025)^2 \qquad 4\text{-}53$$

or

$$\Delta b = 0.05. \qquad 4\text{-}54$$

This then is the maximum permissible value of susceptance modulation amplitude corresponding to a maximum permissible VSWR of 1.05 at match. Note that this is the same as the value determined in section 7 of Chapter III.

The next step in the analysis is to examine the effect of the susceptance modulation at conditions other than matched. First consider the non-time varying term of K_t^2. From equation 3-40

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$$K_{t_o}^2 = \frac{4K_L^2 + N_o'}{4 + N_o'} \qquad 4\text{-}55$$

$$= \frac{4K_L^2 - 4K_L\bar{b}_s \sin 2\theta + (1+K_L^2 - 2K_L\cos 2\theta)\left(\bar{b}_s^2 + \frac{(\Delta b)^2}{2}\right)}{4 - 4K_L\bar{b}_s \sin 2\theta + (1+K_L^2 - 2K_L\cos 2\theta)\left(\bar{b}_s^2 + \frac{(\Delta b)^2}{2}\right)}. \qquad 4\text{-}56$$

This term has its minimum value at match, as determined by equation 4-50; and its maximum value is very nearly K_L, occuring when the average value of stub susceptance is zero. Examining equation 4-56 the following facts become evident:

(a) The magnitude increases as K_L increases.

(b) The magnitude decreases as $\bar{b}_s$ increases up to the match value of stub susceptance; beyond this value, the magnitude again increases; provided 2θ is not equal to zero degrees or 180 degrees.

(c) There is a value of 2θ at which the expression becomes zero (disregarding the component due to $(\Delta b)^2$) if $\bar{b}_s$ has the match value. This value is not apparent from an inspection of the equation; it is of course the value of 2θ given by equation 4-19.

In equation 4-56, the time varying components of the denominator of equation 4-40 have been disregarded because they do not affect the non-time varying component of the square of the reflection coefficient.

Now examine the time varying component of equation 4-40 which is at the fundamental modulation frequency:

$$K_{t_1}^2 = \frac{N_1' \cos\omega_m t}{4+N_0' + N_1'\cos\omega_m t} = \frac{N_1''}{D_1''}\cos\omega_m t. \qquad 4\text{-}57$$

The numerator and denominator of this expression are:

$$N_1'' = \Delta b\left[2\bar{b}_s(1+K_L^2-2K_L\cos 2\theta) - 4K_L\sin 2\theta\right] \qquad 4\text{-}58$$

$$D_1'' = 4 - 4K_L\bar{b}_s\sin 2\theta + (1+K_L^2-2K_L\cos 2\theta)\left(\bar{b}_s^2 + \frac{(\Delta b)^2}{2}\right)$$
$$+\Delta b\left[2\bar{b}_s(1+K_L^2-2K_L\cos 2\theta) - 4K_L\sin 2\theta\right]\cos\omega_m t. \qquad 4\text{-}59$$

Dividing these expressions by the quantity $2(1+K_L^2-2K_L\cos 2\theta)$ they become

$$N_1'' = \Delta b\left[\bar{b}_s - \frac{2K_L\sin 2\theta}{1+K_L^2-2K_L\cos 2\theta}\right] \qquad 4\text{-}60$$

$$D_1'' = \frac{2}{1+K_L^2-2K_L\cos 2\theta} - \frac{2K_L\sin 2\theta}{1+K_L^2-2K_L\cos 2\theta}\bar{b}_s$$
$$+\frac{1}{2}\left[\bar{b}_s^2 + \frac{(\Delta b)^2}{2}\right] + \Delta b\left[\bar{b}_s - \frac{2K_L\sin 2\theta}{1+K_L^2-2K_L\cos 2\theta}\right]\cos\omega_m t. \qquad 4\text{-}61$$

And substituting from equation 4-15, the numerator and denominator of 4-57 become,

$$N_1'' = \Delta b\left[\bar{b}_s + b_\ell\right] \qquad 4\text{-}62$$

$$D_1'' = \frac{2}{1+K_L^2-2K_L\cos 2\theta} - \bar{b}_s b_\ell + \frac{1}{2}\left[\bar{b}_s^2 + \frac{(\Delta b)^2}{2}\right] + \Delta b\left[\bar{b}_s + b_\ell\right]\cos\omega_m t. \qquad 4\text{-}63$$

So that the fundamental frequency time varying component of the square of the reflection coefficient becomes,

$$K_{t_1}^2 = \frac{\Delta b\left[\bar{b}_s + b_\ell\right]\cos\omega_m t}{\frac{2}{1+K_L^2-2K_L\cos 2\theta} - \bar{b}_s b_\ell + \frac{1}{2}\left[\bar{b}_s^2 + \frac{(\Delta b)^2}{2}\right] + \Delta b\left[\bar{b}_s + b_\ell\right]\cos\omega_m t}. \qquad 4\text{-}64$$

This expression is zero whenever the term $(b_s + b_\ell)$ is zero; that is, whenever the stub susceptance is equal to the magnitude of the line susceptance so that the total input susceptance is zero. The expression is negative when the stub susceptance is greater than the magnitude of the line susceptance, and is positive when the stub susceptance is less than the magnitude of the line susceptance; thus, a change of sign takes place whenever the expression passes through zero, and this constitutes a phase reversal that provides the directional sense. As was noted in Chapter III, the fundamental frequency time varying component is zero whenever the total input susceptance is zero, regardless of the position of the stub. The magnitude of this term is very nearly directly proportional to the amplitude of susceptance modulation, and also very nearly directly proportional to the value of the total input susceptance of the system, so that the amplitude of the signal decreases as the stub susceptance approaches the magnitude of the line susceptance.

The second harmonic time varying component of the square of the total reflection coefficient is given by

$$K_{t_2}^2 = \frac{N_2' \cos 2\omega_m t}{4 + N_0' + N_1' \cos \omega_m t + N_2' \cos 2\omega_m t} \qquad 4\text{-}65$$

where

$$N_2' = \frac{(\Delta b)^2}{2}\left(1 + K_L^2 - 2K_L \cos 2\theta\right). \qquad 4\text{-}66$$

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It is apparent that the amplitude of this term is always small, and is small compared to the amplitude of the fundamental component, except when the total input susceptance is at, or very nearly at, zero. The second harmonic component is present at the match condition as was shown previously, and is the only time varying component present whenever the total input susceptance becomes zero.

5. Analysis of the position modulation.

The analysis of the effect of position modulation on the signal will proceed in the same manner as the susceptance modulation. Rewriting equations 4-29 and 4-30

$$K_t^2 = \frac{4K_L^2 + b_s^2(1+K_L^2 - 2K_L\cos 2\theta) - 4K_L \sin 2\theta}{4 + b_s^2(1+K_L^2 - 2K_L\cos 2\theta) - 4K_L \sin 2\theta} \qquad \text{4-29}$$

$$\theta = \bar{\theta} + \Delta\theta \sin \omega_m t. \qquad \text{4-30}$$

Rearranging the numerator of equation 3-29 to a form more suitable for the substitution of equation 3-30

$$N = 4K_L^2 + b_s^2(1+K_L^2 - 2K_L\cos 2\theta) - 4K_L b_s \sin 2\theta \qquad \text{4-66}$$

$$= 4K_L^2 + b_s^2 + K_L^2 b_s^2 - 2K_L b_s(b_s\cos 2\theta + 2\sin 2\theta) \qquad \text{4-67}$$

$$= 4K_L^2 + b_s^2(1+K_L^2) - 2K_L b_s^2 \cos 2\theta - 4K_L b_s \sin 2\theta \qquad \text{4-68}$$

$$= A + B\cos 2\theta + C\sin 2\theta. \qquad \text{4-69}$$

And substituting from equation 3-30

$$N = A + B\cos 2(\bar{\theta} + \Delta\theta \sin \omega_m t)$$

$$+ C \sin 2(\bar{\theta} + \Delta\theta \sin \omega_m t) \qquad 4\text{-}70$$

$$= A + B\left[\cos 2\bar{\theta} \cos(2\Delta\theta \sin\omega_m t) - \sin 2\theta \sin(2\Delta\theta \sin\omega_m t)\right]$$

$$+ C\left[\sin 2\bar{\theta} \cos(2\Delta\theta \sin\omega_m t) + \cos 2\bar{\theta} \cos(2\Delta\theta \sin\omega_m t)\right] \qquad 4\text{-}71$$

$$= A + \left[B\cos 2\bar{\theta} + C \sin 2\bar{\theta}\right]\cos(2\Delta\theta \sin\omega_m t)$$

$$- \left[B\sin 2\bar{\theta} - C\cos 2\bar{\theta}\right] \sin(2\Delta\theta \sin\omega_m t). \qquad 4\text{-}72$$

Approximating:

$$\cos(2\Delta\theta \sin\omega_m t) \doteq 1 - 2(\Delta\theta)^2 \sin^2\omega_m t, \qquad 4\text{-}73$$

$$\sin(2\Delta\theta \sin\omega_m t) \doteq 2\Delta\theta \sin\omega_m t. \qquad 4\text{-}74$$

These approximations are well founded if $\Delta\theta$ is small; that $\Delta\theta$ must be small is apparent from the explanation of Chapter III, and will be shown analytically below. Substituting from equations 4-73 and 4-74

$$N = A + \left[B\cos 2\bar{\theta} + C\sin 2\bar{\theta}\right]\left[1 - 2(\Delta\theta)^2 \sin^2\omega_m t\right]$$

$$- \left[B\sin 2\bar{\theta} - C\cos 2\bar{\theta}\right] 2\Delta\theta \sin\omega_m t \qquad 4\text{-}75$$

$$= A + B\cos 2\bar{\theta} + C\sin 2\bar{\theta}$$

$$- 2(\Delta\theta)^2\left[B\cos 2\bar{\theta} + C\sin 2\bar{\theta}\right] \frac{1 - \cos 2\omega_m t}{2}$$

$$- 2\Delta\theta\left[B\sin 2\bar{\theta} - C\cos 2\bar{\theta}\right] \sin\omega_m t \qquad 4\text{-}76$$

$$N = A + \left[1-(\Delta\Theta)^2\right](B\cos 2\bar{\Theta} + C\sin 2\bar{\Theta})$$

$$-2\Delta\Theta(B\sin 2\bar{\Theta} - C\cos 2\bar{\Theta})\sin\omega_m t$$

$$+ (\Delta\Theta)^2(B\cos 2\bar{\Theta} + C\sin 2\bar{\Theta})\cos 2\omega_m t \qquad 4\text{-}77$$

$$= N_0' + N_1'\sin\omega_m t + N_2'\cos 2\omega_m t\ , \qquad 4\text{-}78$$

where

$$N_0' = A + \left[1-(\Delta\Theta)^2\right](B\cos 2\bar{\Theta} + C\sin 2\bar{\Theta}) \qquad 4\text{-}79$$

$$= 4K_L^2 + b_s^2(1+K_L^2) + \left[1-(\Delta\Theta)^2\right](-2K_L b_s^2\cos 2\bar{\Theta} - 4K_L b_s\sin 2\bar{\Theta}) \qquad 4\text{-}80$$

$$= 4K_L^2 + b_s^2(1+K_L^2 - 2K_L\cos 2\bar{\Theta}) - 4K_L b_s\sin 2\bar{\Theta}$$

$$+2K_L b_s(\Delta\Theta)^2(b_s\cos 2\bar{\Theta} + 2\sin 2\bar{\Theta})\ , \qquad 4\text{-}81$$

$$N_1' = -2\Delta\Theta(-2K_L b_s^2\sin 2\bar{\Theta} + 4K_L b_s\cos 2\bar{\Theta}) \qquad 4\text{-}82$$

$$= 4\Delta\Theta K_L b_s(b_s\sin 2\bar{\Theta} - 2\cos 2\bar{\Theta})\ , \qquad 4\text{-}83$$

$$N_2' = (\Delta\Theta)^2(-2K_L b_s^2\cos 2\bar{\Theta} - 4K_L b_s\sin 2\bar{\Theta}) \qquad 4\text{-}84$$

$$= -2(\Delta\Theta)^2 K_L b_s(b_s\cos 2\bar{\Theta} + 2\sin 2\bar{\Theta})\ . \qquad 4\text{-}85$$

Similarly,

$$D_0' = 4 + b_s(1+K_L^2 - 2K_L\cos 2\bar{\Theta}) - 4K_L b_s\sin 2\bar{\Theta}$$

$$+ 2K_L b_s(\Delta\Theta)^2(b_s\cos 2\bar{\Theta} + 2\sin 2\bar{\Theta})\ , \qquad 4\text{-}86$$

$$D_1' = N_1'\ , \qquad 4\text{-}87$$

$$D_2' = N_2'\ . \qquad 4\text{-}88$$

So that

$$K_t^2 = \frac{N_0' + N_1' \sin \omega_m t + N_2' \cos 2\omega_m t}{D_0' + N_1' \sin \omega_m t + N_2' \cos 2\omega_m t} . \qquad 4\text{-}89$$

This is the complete expression showing the effect of the position modulation. First examine equation 4-89 at match. To do this, substitute equations 4-19 and 4-20 in equations 4-81, 4-83, 4-85 and 4-86, obtaining for N_0'

$$N_0' = 4K_L^2 + \frac{4K_L^2}{1-K_L^2}(1-K_L^2) - 4K_L \frac{2K_L}{\sqrt{1-K_L^2}}\sqrt{1-K_L^2}$$
$$+ 2K_L \frac{2K_L}{\sqrt{1-K_L^2}} (\Delta\theta)^2 \left(\frac{2K_L}{\sqrt{1-K_L^2}} K_L + 2\sqrt{1-K_L^2}\right) \qquad 4\text{-}90$$
$$= 8K_L^2 (\Delta\theta)^2 \left(\frac{K_L^2}{1-K_L^2} + 2\right) , \qquad 4\text{-}91$$

and for N_1'

$$N_1' = 4\Delta\theta K_L \frac{2K_L}{\sqrt{1-K_L^2}} \left(\frac{2K_L}{\sqrt{1-K_L^2}}\sqrt{1-K_L^2} - 2K_L\right) = 0 , \qquad 4\text{-}92$$

and for N_2'

$$N_2' = 8K_L^2 (\Delta\theta)^2 \left(\frac{K_L^2}{1+K_L^2} + 2\right) , \qquad 4\text{-}93$$

and for D_0'

$$D_0' = 4 - 4K_L^2 + 8K_L^2(\Delta\theta)^2\left(\frac{K_L^2}{1-K_L^2} + 2\right) . \qquad 4\text{-}94$$

Then at the match conditions

$$K_{t_M}^2 = \frac{8K_L^2(\Delta\theta)^2\left(\frac{K_L^2}{1-K_L^2} + 2\right)(1+\cos 2\omega_m t)}{4 - 4K_L^2 + 8K_L^2(\Delta\theta)^2\left(\frac{K_L^2}{1-K_L^2} + 2\right)(1+\cos 2\omega_m t)} \qquad 4\text{-}95$$

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$$K_{t_M}^2 = \frac{8K_L^2(\Delta\Theta)^2\left[K_L^2+2(1-K_L^2)\right](1+\cos 2\omega_m t)}{4(1-K_L^2)-4K_L^2(1-K_L^2)+8K_L^2(\Delta\Theta)^2\left[K_L^2+2(1-K_L^2)\right](1+\cos 2\omega_m t)} \qquad 4\text{-}96$$

$$= \frac{2K_L^2(\Delta\Theta)^2(1+\cos 2\omega_m t)}{1-K_L^2+2K_L^2(\Delta\Theta)^2(1+\cos 2\omega_m t)} \, . \qquad 4\text{-}97$$

From the non-time varying component, the maximum permissible amplitude of position modulation may be determined.

$$K_{t_{M_0}}^2 = \frac{2K_L^2(\Delta\Theta)^2}{1-K_L^2+2K_L^2(\Delta\Theta)^2} \, . \qquad 4\text{-}98$$

It is apparent from this expression that for a specified value of the square of the total reflection coefficient the maximum permissible amplitude of position modulation is inversely dependent on the magnitude of the load reflection coefficient. Or, in a different manner of speaking, the non-time varying component is directly proportional to the square of the position modulation amplitude, and is very nearly directly proportional to the square of the load reflection coefficient. Thus if a maximum permissible value of position modulation amplitude is to be specified, the magnitude of the load reflection coefficient must also be specified; a value of load VSWR of 4.0 is a reasonable maximum value to choose, since it is probably greater than would normally be encountered in a transmission system. This

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corresponds to a value of 0.6 for the magnitude of the load reflection coefficient. Again specifying the maximum permissible value of total reflection coefficient as 0.025 as was done in the case of the susceptance modulation, then

$$(0.025)^2 = \frac{2 \times 0.36(\Delta\theta)^2}{1-0.36+2\times 0.36(\Delta\theta)^2}, \tag{4-99}$$

and if $\Delta\theta$ is small, then $0.72(\Delta\theta)^2$ is very small compared with 0.64, so that an approximation may be made, and

$$(0.025)^2 \doteq \frac{2\times 0.36(\Delta\theta)^2}{0.64}, \tag{4-100}$$

$$(\Delta\theta)^2 \doteq \frac{0.64}{0.72}\times(0.025)^2, \tag{4-101}$$

$$\Delta\theta \doteq 0.0236^r = 0.00376\lambda = 1.35^\circ. \tag{4-102}$$

While this value of position modulation amplitude will provide a very small VSWR for large values of load VSWR, the fundamental frequency component will be shown to be proportional to the value of position modulation amplitude, and a larger value of position modulation amplitude may be required in practice in order to provide an adequate signal when the load VSWR is small.

From equation 4-97 it is apparent that at match there is no component at the fundamental modulation frequency. The average value of the reflection coefficient is very small and is given approximately by

$$K_{t_M}^2 \doteq \frac{2K_L^2(\Delta\theta)^2}{1-K_L^2}. \tag{4-103}$$

There is a signal at double the fundamental modulation frequency with approximately the same magnitude as the non-time varying component. Unlike the match condition with respect to susceptance modulation, K_{tM}^2 is not independent of all the parameters of the system with respect to the position modulation, but depends on the magnitude of the load reflection coefficient as well as on the amplitude of position modulation.

The next step in the analysis is to examine the effect of the position modulation at conditions other than matched. First consider the non-time varying term of K_t^2. From equation 4-89

$$K_{t_0}^2 = \frac{N_0'}{-D_0'} \qquad 4\text{-}104$$

and from equations 4-80 and 4-86,

$$K_{t_0}^2 = \frac{4K_L^2 + b_s^2(1+K_L^2) - 2K_L b_s\left[1-(\Delta\theta)^2\right](b_s\cos 2\bar{\theta} - 2\sin 2\bar{\theta})}{4 + b_s^2(1+K_L^2) - 2K_L b_s\left[1-(\Delta\theta)^2\right](b_s\cos 2\bar{\theta} - 2\sin 2\bar{\theta})} \qquad 4\text{-}105$$

This term has its minimum value at match, as determined by equation 4-103; and its maximum value is K_L, occuring when the stub susceptance is zero. The following effects are also evident:

(a) The magnitude increases as K_L increases.

(b) Depending on the value of $2\bar{\theta}$, the magnitude may either decrease or increase as the stub susceptance increases.

(c) There is a value of $2\bar{\theta}$ at which the expression becomes zero when the stub susceptance has the

match value. While it is not apparent from an inspection of equation 4-105, this value of $2\bar{\Theta}$ is that given by equation 4-16.

In equation 4-105 the time varying components of the denominator of equation 4-89 have been disregarded because they do not affect the non-time varying component of the square of the reflection coefficient.

Now examine the fundamental modulation frequency component of equation 4-89. This is given by

$$K_{t_1}^2 = \frac{N_1' \sin \omega_m t}{D_0' + N_1' \sin \omega_m t} \qquad \text{4-106}$$

$$= \frac{4\Delta\Theta\, K_L b_s (b_s \sin 2\bar{\Theta} - 2\cos 2\bar{\Theta}) \sin \omega_m t}{D_0' + 4\Delta\Theta\, K_L b_s (b_s \sin 2\bar{\Theta} - 2\cos 2\bar{\Theta}) \sin \omega_m t}. \qquad \text{4-107}$$

This term is zero whenever the term

$$b_s \sin 2\bar{\Theta} - 2\cos 2\bar{\Theta} \qquad \text{4-108}$$

is zero; setting this term equal to zero and solving yields

$$\tan 2\bar{\Theta} = \frac{2}{b_s}. \qquad \text{4-109}$$

By substituting from equations 4-16 and 4-20, it is seen that 4-109 is satisfied at the match condition.

$$\frac{\sqrt{1-K_L^2}}{K_L} \equiv \frac{2}{2K_L/\sqrt{1-K_L^2}}. \qquad \text{4-10}$$

Furthermore, equation 4-109 can be used to determine the loci of total input admittance which yield no positioning

signal; in fact, equation 4-109 was used to compute the loci of Figure 12, page 45, of Chapter III. Equation 4-109 determines the values of $2\bar{\theta}$ which the stub seeks and will hold, in terms of the average value of b_s, and as pointed out in Chapter II, the stub will not seek and hold the match position until the stub susceptance reaches the match value. It should be noted from equation 4-107 that the positioning signal is directly proportional to the amplitude of position modulation and to the magnitude of the load reflection coefficient, and is proportional to the value of b_s, the stub susceptance. The proportionality simultaneously to K_L and position modulation amplitude was previously mentioned in connection with the average magnitude of the reflection coefficient at match; and it is now seen that for small values of load VSWR a larger value of position amplitude modulation will be required than for large values of load VSWR. Thus if a relatively large value of position modulation amplitude is provided, large load VSWRs will result in a relatively large average reflection coefficient at match; while if a relatively small value of position modulation amplitude is provided, insufficient signal may be available when the load VSWR is low. This means that in a practical system, the amplitude of position modulation must be determined by the values of load VSWR expected.

Now examine the component at double the modulation frequency given by

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$$K_{t_2}^2 = \frac{N_2' \cos 2\omega_m t}{D_0' + N_1' \sin \omega_m t + N_2' \cos 2\omega_m t} \qquad 4\text{-}111$$

$$= \frac{-2K_L b_s (\Delta\theta)^2 (b_s \cos 2\bar{\theta} + 2\sin 2\bar{\theta}) \cos 2\omega_m t}{D_0' + N_1' \sin \omega_m t - 2K_L b_s (\Delta\theta)^2 (b_s \cos 2\bar{\theta} + 2\sin 2\bar{\theta}) \cos 2\omega_m t} \,. \qquad 4\text{-}112$$

This term, like the double frequency susceptance modulation term is very small, and is small compared to the fundamental frequency except when the stub is at or very near the stable no-signal position; that is, when $2\bar{\theta}$ is at or very near the value determined by equation 4-109. Like the fundamental position modulation term it is proportional to the magnitude of the load reflection coefficient and nearly proportional to the stub susceptance. Equation 4-112 indicates that there is a position at which the double frequency term becomes zero; this is the result of the approximation and is not necessarily true.

As a final note it should be mentioned that the approximations made in the solution of position modulation led to the discarding of higher order harmonic components which are actually present but have extremely small amplitude.

6. Cross-talk between the two signals.

(a) Cross-talk in the susceptance signal from position modulation. This type of cross-talk may be investigated by examining the expression for the fundamental frequency time varying component of the square of the reflection coefficient due to susceptance modulation, equation 4-64:

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$$K_{t_1}^2 = \frac{\Delta b(\bar{b}_s + b_\ell)\cos\omega_m t}{\frac{2}{1+K_L^2-2K_L\cos 2\theta} - \bar{b}_s b_\ell + \frac{1}{2}\left[\bar{b}_s^2 + \frac{(\Delta b)^2}{2}\right] + \Delta b(\bar{b}_s + b_\ell)\cos\omega_m t} \quad 4\text{-}64$$

The principle source of cross-talk in this expression is contained in the numerator in the value of the line susceptance, since the value of the line susceptance is modulated by the position modulation. The line susceptance is given by equation 4-17

$$b_\ell = \frac{-2K_L \sin 2\theta}{1+K_L^2 - 2K_L\cos 2\theta} . \quad 4\text{-}17$$

Substituting equation 4-30 into equation 4-17, the line susceptance becomes

$$b_\ell = \frac{-2K_L \sin 2(\bar{\theta} + \Delta\theta \sin\omega_m t)}{1+K_L^2 - 2K_L\cos 2(\bar{\theta} + \Delta\theta\sin\omega_m t)} \quad 4\text{-}113$$

And thus the numerator of equation 4-64 becomes

$$N_1 = \Delta b\left[\bar{b}_s - \frac{2K_L \sin 2(\bar{\theta} + \Delta\theta\sin\omega_m t)}{1+K_L^2 - 2K_L\cos 2(\bar{\theta} + \Delta\theta\sin\omega_m t)}\right]\cos\omega_m t . \quad 4\text{-}114$$

It is apparent from this expression that the position modulation cannot introduce any cross-talk into the susceptance signal other than second order effects. These effects will be negligably small except when the stub susceptance is equal to the magnitude of the line susceptance; even then the cross-

talk will be 90 degrees out of phase with the susceptance signal.

(b) Cross-talk in the position signal from susceptance modulation. This type of cross-talk may be investigated by examining the expression for the fundamental frequency time varying component of the square of the reflection coefficient due to position modulation, equation 4-107:

$$K_{t_1}^2 = \frac{4\Delta\theta K_L b_s (b_s \sin 2\bar{\theta} - 2\cos 2\bar{\theta}) \sin\omega_m t}{D_0' + 4\Delta\theta K_L b_s (b_s \sin 2\bar{\theta} - 2\cos 2\bar{\theta}) \sin\omega_m t} \qquad 4\text{-}107$$

The principle source of cross-talk in this expression is contained in the numerator in the instantaneous value of stub susceptance, b_s. The instantaneous value of stub susceptance is given by equation 3-2:

$$b_s = \bar{b}_s + \Delta b \cos\omega_m t \qquad 3\text{-}2$$

Substituting this expression into the numerator of equation 4-107 yields

$$N_1 = 4\Delta\theta K_L (\bar{b}_s + \Delta b \cos\omega_m t)\left[(\bar{b}_s + \Delta b \cos\omega_m t)\sin 2\bar{\theta} - 2\cos 2\bar{\theta}\right]\sin\omega_m t \qquad 4\text{-}115$$

And again it is apparent that the cross-talk will have only second order effects.

Since the proposed system includes filters to reject harmonic components of the signals it is unnecessary to examine the cross-talk contained in the harmonics.

7. Conclusion.

The mathematical analysis of this chapter has led to expressions for the two positioning signals which verify the results of the intuitive investigation of Chapter II and the qualitative analysis of Chapter III. Only the relative magnitudes of the signals have been discussed, since the absolute magnitudes will depend on the power level at which the transmission system is operating. Thus, each different power level will require different amplitudes of modulation to develop signals of sufficient amplitude to drive the servo amplifiers. It is apparent that practically speaking, the most satisfactory method of determining the amplitudes of modulation in any given system will be an experimental method. Ultimately, the minimum amplitudes of modulation will be determined, in a given system operating at a given power level, by the noise level and gain of the amplifier. Thus in any case, the amplifier should have the lowest possible noise level and a gain sufficient to drive the servo motors with input signals of the same order of magnitude as the noise of the first stage.

CHAPTER V

THE SUSCEPTANCE PROBE TUNER

1. Introduction.

The Susceptance Probe Tuner is an automatic impedance matching device designed to operate in a waveguide transmission system in the frequency band 8200 to 9600 megacycles per second. This tuner was proposed by Walters [14], and initial investigations were made by Red [11]. The single movable shunt susceptance is in the form of a capacitive probe which is inserted into the waveguide at the center of a broad face. In order to adjust the magnitude of the susceptance, the depth of penetration is variable and controlled by a servo loop. The position of the probe is variable through a distance of one half wavelength along the waveguide and is controlled by a servo loop. Susceptance modulation is achieved by means of a small auxiliary probe, placed at the same position as the susceptance probe, in the opposite face of the waveguide. Position modulation is achieved by varying the electrical separation of the load and the susceptance probe by means of a small dielectric card phase shifter. The reflected wave is detected by means of a directional coupler and crystal detector. Each servo loop consists of a preamplifier, filter networks, phase sensitive rectifiers, power amplifier, drive motor, and drive mechanism. The modulation drive system consists

of a drive motor and mechanism, both modulations being driven by the same motor. The modulation drive motor also drives a two phase spin generator at the modulation frequency to supply the reference phases for the phase sensitive rectifiers. Figure 15 is a block diagram of the Susceptance Probe Tuner, and Figure 16 is a photograph of the system. The directional coupler, crystal mount, and system termination are not shown in the photograph.

2. Characteristics of the susceptance probe.

Ideally, the capacitive susceptance probe utilized in this system is capable of presenting all values of positive susceptance from zero at zero insertion into the waveguide to infinity at an insertion of one quarter wavelength, and presents zero conductance at all depths of penetration. In the case of the screw type susceptance probe frequently used to reduce the VSWR in a waveguide transmission system, this ideal performance can be very nearly attained by using a material with low surface resistivity and using the threads on the screw to insure a good electrical path between the screw and the waveguide. In the Susceptance Probe Tuner, the requirement that the probe be capable of moving along the waveguide complicates the provision of a low impedance path between the probe and the waveguide; it is necessary to use a system of choke joint and quarter wavelength coaxial line to provide this low impedance path. This system of course imposes a bandwidth limitation on the Tuner, but if the choke

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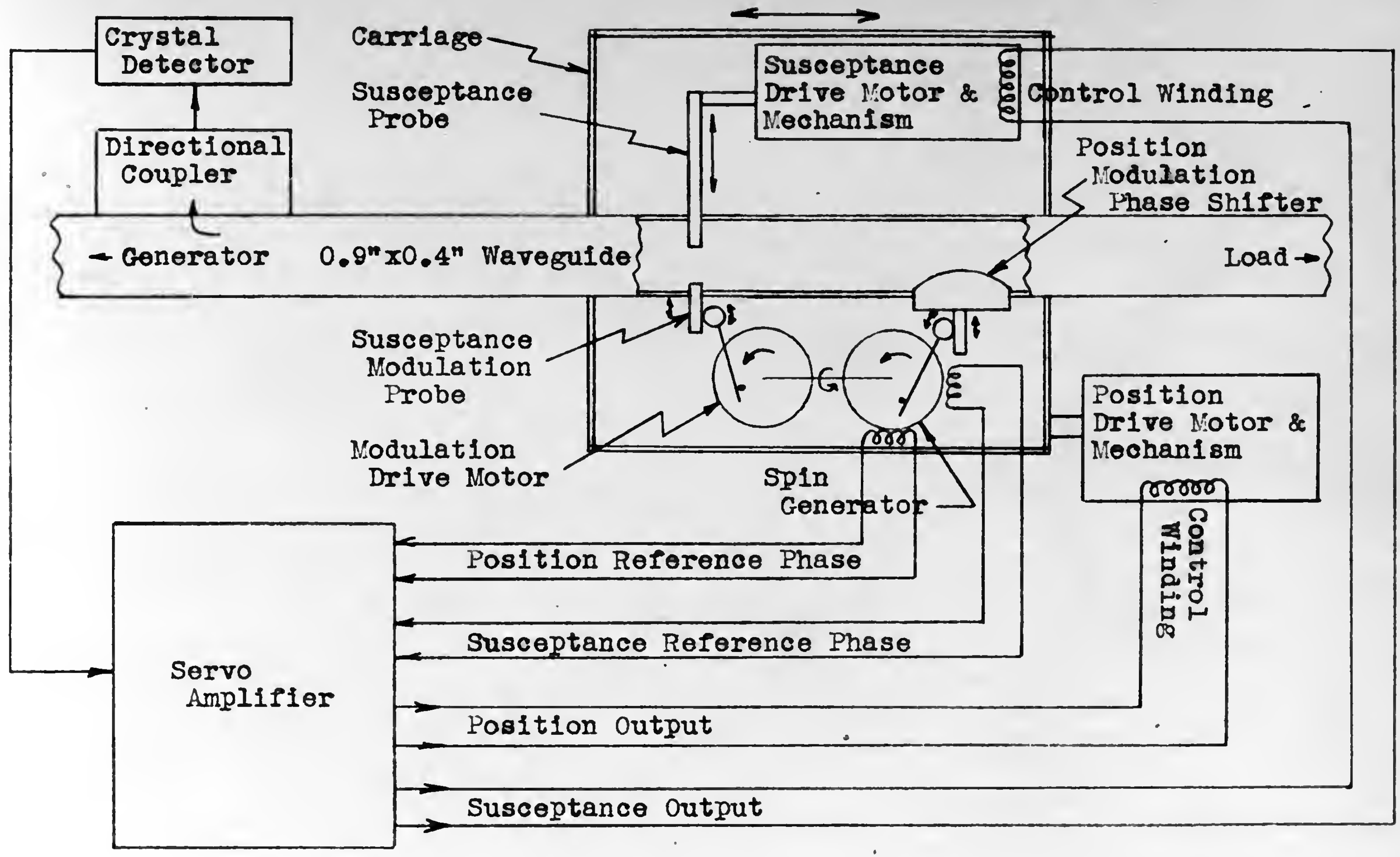


Figure 15

Block Diagram of the Susceptance Probe Tuner

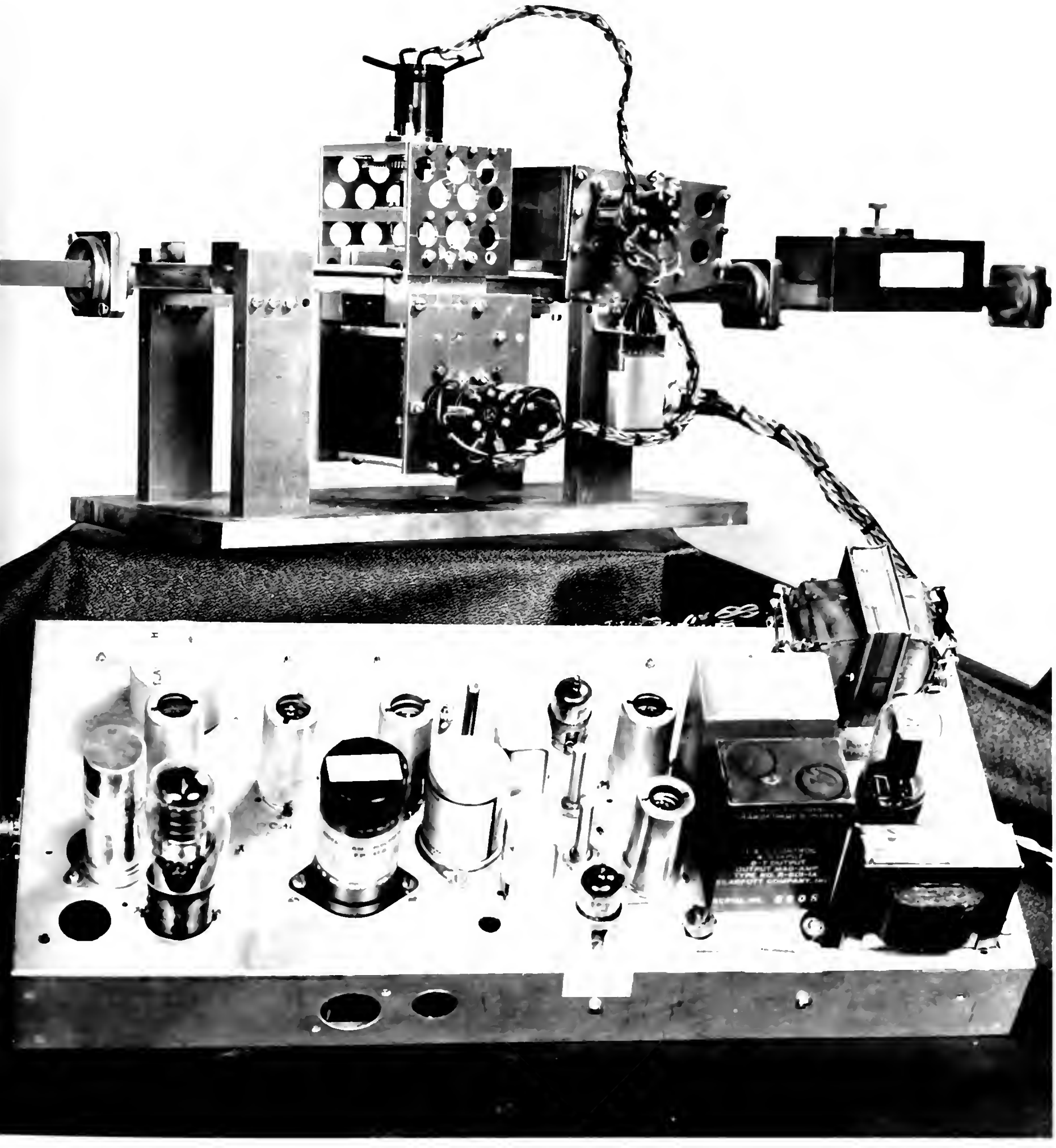


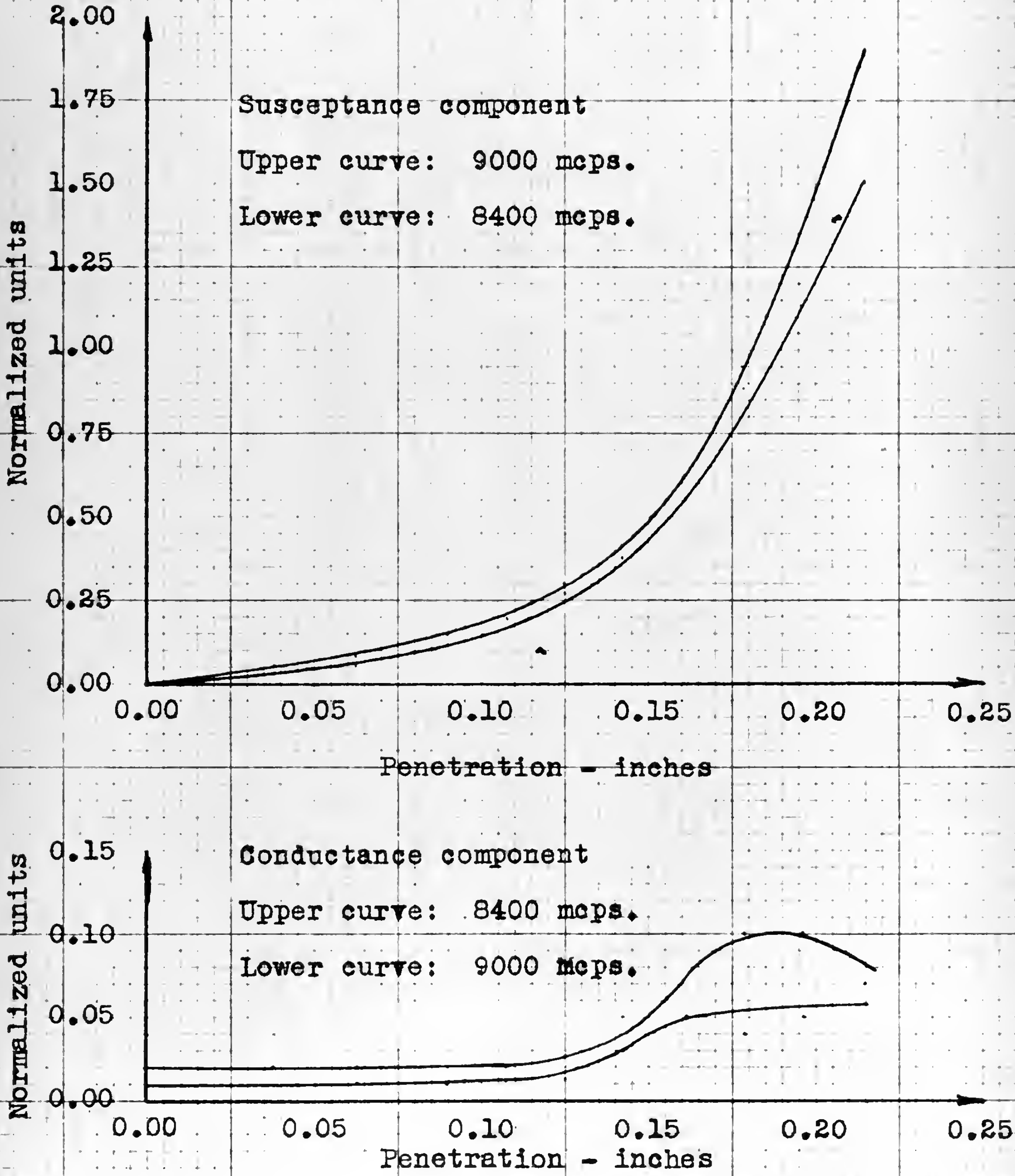
Figure 16

The Susceptance Probe Tuner Mechanism and Servo Amplifier

joint is properly designed for wide bandwidth the limitation need not be severe. The characteristics of the probe and mount used in the Susceptance Probe Tuner are shown in Figure 17, and it is to be noted that the probe exhibits only a very small conductance component even at depths of penetration approaching a quarter wavelength. This low conductance component is indicative of low loss, one of the desirable characteristics outlined in Chapter I.

Figure 18 is a drawing of the probe mount incorporating the choke joint and quarter wavelength coaxial line. The coaxial line formed by the probe and choke joint body has a very low characteristic impedance due to the large radius ratio. Since the coaxial line is approximately open circuited at its outer end and is approximately one quarter wavelength long in the operating frequency band, the impedance seen at the inner end is very low. The choke joint then provides a low impedance path from the inner end of the coaxial line to the waveguide surface; thus resulting in a low impedance path from probe to waveguide. The effectiveness of this mount is well illustrated by the nearly ideal performance of the probe shown in Figure 17. The klystron used to determine these characteristics had a lower frequency limit of 8200 megacycles per second, and operation of the probe was satisfactory at this lower limit. The probe had an upper frequency limit of 9400 megacycles per second. Time did not permit attempting to extend the upper frequency limit of

Figure 17



Admittance of the Susceptance Probe

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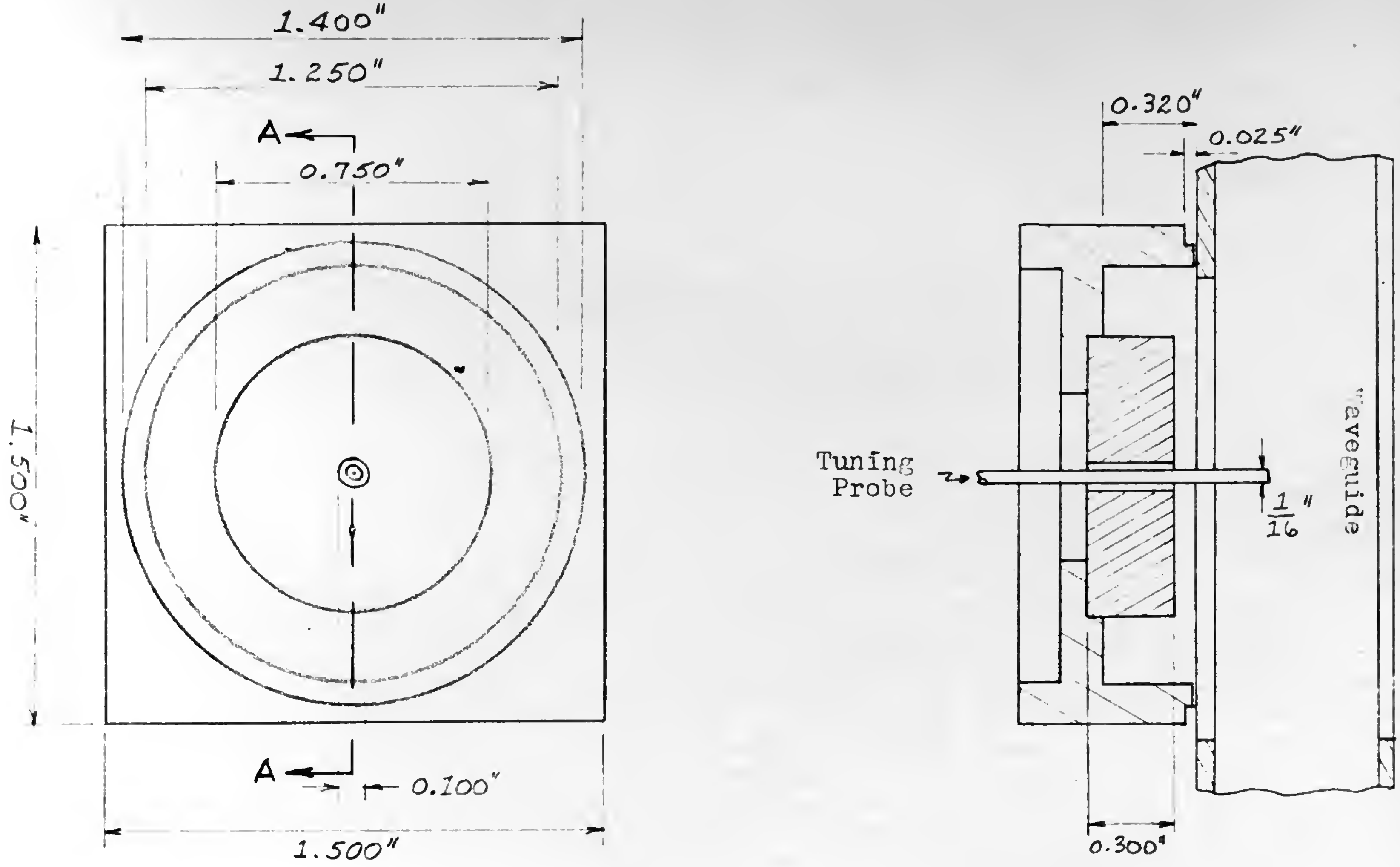


Bottom View

Section A-A

Figure 18

Drawing of the Choke Joint Probe Mount

satisfactory operation, although it is felt that this would be quite easily done. The present upper frequency limit is undoubtedly due to the frequency sensitive nature of the probe mount.

3. Susceptance modulation.

The susceptance modulation probe is a small auxiliary probe penetrating a short distance into the waveguide directly opposite the susceptance probe. In this position, the susceptance of the modulation probe is effectively in parallel with the susceptance of the susceptance probe in the manner proposed in Chapter III, section 2, page 32. The modulation probe is supported by the same carriage as the susceptance so that they move longitudinally along the waveguide together. The depth of penetration of the modulation probe is varied over a small amplitude about the mean penetration by the modulation drive motor and mechanism. In terms of equation 3-2 then,

$$b_s = \overline{b_s} + \Delta b \cos \omega_m t, \qquad 3\text{-}2$$

b_s is the instantaneous sum of the susceptance of the susceptance probe and the susceptance of the modulation probe; $\overline{b_s}$ is the sum of the susceptance of the susceptance probe and the average value of susceptance of the modulation probe; and Δb is the amplitude of variation of modulation probe susceptance. If the susceptance probe has zero penetration into the waveguide, the average value of modulation probe susceptance provides the small susceptance value required at all times

for proper position signal generation. (See Chapter III, section 4, page 37). In testing the system it was found that satisfactory susceptance signals were developed with the average value of modulation probe susceptance set at 0.05 and the amplitude of susceptance modulation set at 0.03. This average value of susceptance also provided satisfactory position signals with susceptance probe susceptance at zero. It is to be noted that the susceptance modulation amplitude is less than the maximum permissible values of section 7, Chapter III, page 55, and section 4, Chapter IV, page 68.

4. Position modulation.

To provide position modulation, a dielectric card phase shifter is employed. A thin card of low loss dielectric material (Melamine or Mycalex) penetrates a short distance into the waveguide through the same slot as is used to permit the entry of the susceptance modulation probe. The penetration of the dielectric card is varied with a small amplitude about the average penetration, thus modulating the electrical separation of the load and susceptance probe by modulating the physical length of one wavelength in the waveguide by the expedient of varying the average dielectric constant of the propagating medium. The motion of the dielectric card is mechanically 90 degrees out of phase with the motion of the susceptance modulation probe, thus providing the means of separating the two modulation components through the use of phase sensitive rectifiers in the servo loops. The config-

uration and characteristics of the dielectric card are shown in Figure 19. In testing the system it was found that the range of phase shift in which the VSWR of the dielectric card is less than 1.03 is considerably more than is required to develop an adequate positioning signal, even with values of load VSWR in the vicinity of 1.10. As finally adjusted, the average penetration into the waveguide is about 0.065 inches, and the amplitude of mechanical motion is about 0.035 inches, providing an amplitude of position modulation of about 1.5 electrical degrees. It is to be noted that this value is approximately the same as the maximum permissible value for a load VSWR of 4.0, as determined in section 5, Chapter IV, page 77.

5. The modulation drive system.

Figure 20 is a drawing of the modulation drive system. A single Kearfott Mark 14, Mod. 0, servo motor is used to drive the modulating mechanism and a Kearfott two phase spin generator. The drive mechanisms for the two modulating devices, the susceptance modulation probe and the dielectric card, are identical except in the amplitude of mechanical motion which they provide, and consist of an eccentric driving a lever arm which in turn drives a pinion gear through a small arc. The pinion gear drives a rack mounted on the modulation device support. Phasing is accomplished by separating the two eccentrics 90 degrees mechanically. The mechanical motion given to the modulation devices by this arrangement is not

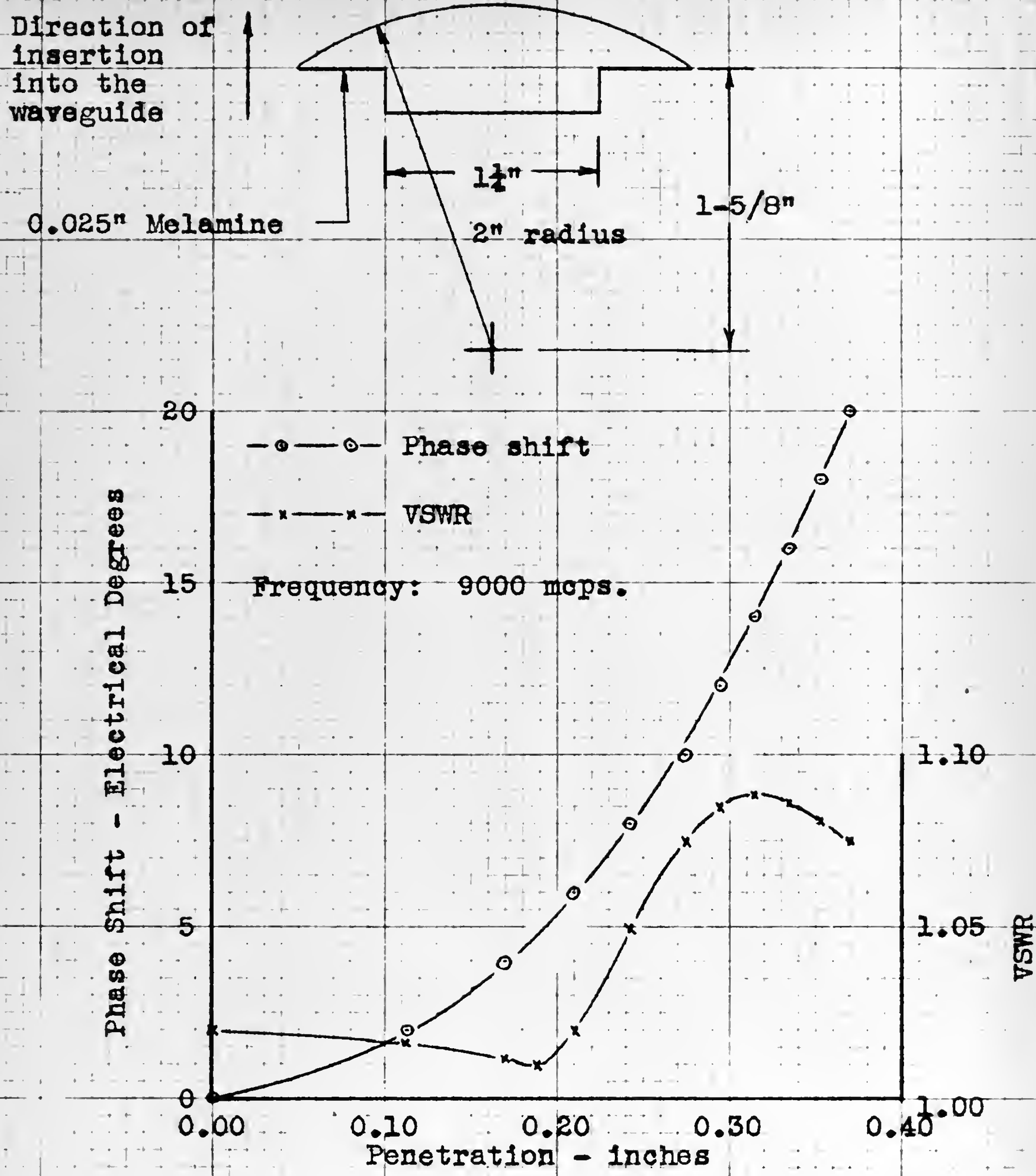


Figure 19

Dielectric Card Phase Shifter

used for position modulation

Configuration and Electrical Characteristics

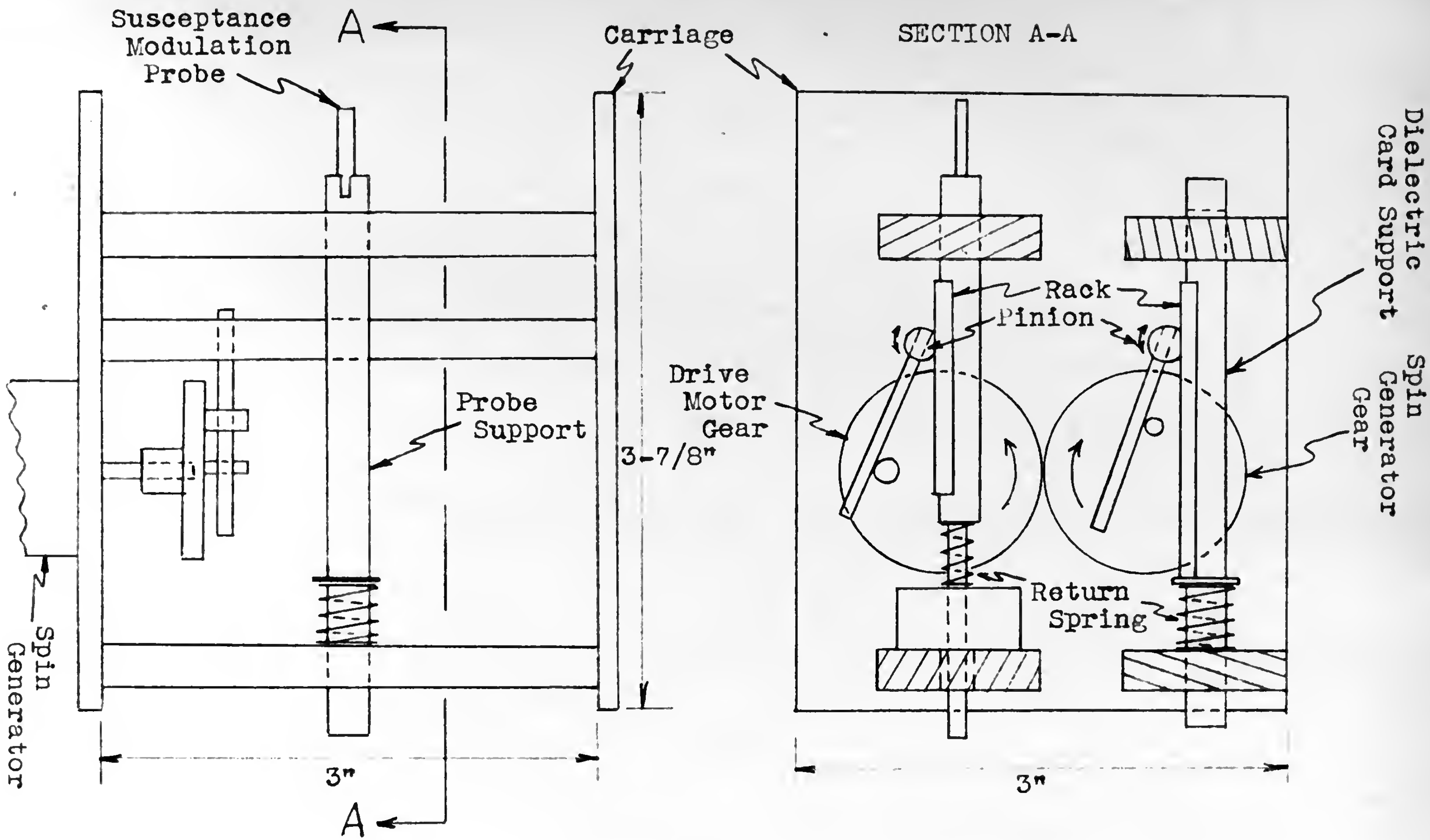


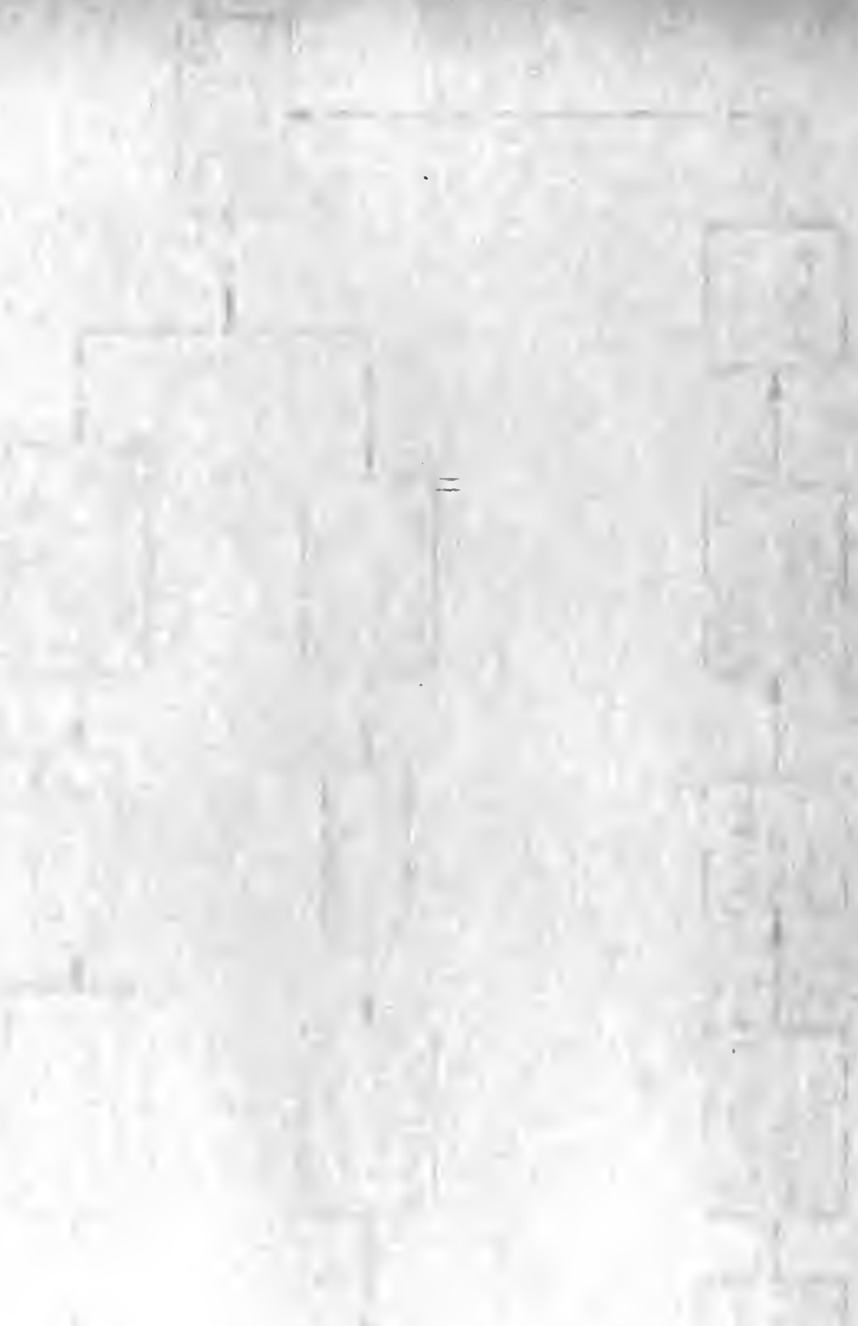
Figure 20

Modulation Drive System

precisely sinusoidal but is completely satisfactory, and the arrangement has the advantage of easy amplitude adjustment by changing the length of the lever arm. This mechanical arrangement yielded a modulation frequency in the range 15 to 50 cycles per second, the particular value being determined by the voltage supplied to the control phase of the servo motor. In testing the system the modulation frequency was finally set to 20 cycles per second, this value being a compromise between smooth mechanical operation and good waveform from the spin generator.

6. The servo amplifier.

The principle special requirement on the amplifier is high sensitivity; this in turn requires a low noise level. Since it is necessary to amplify only the modulation frequency, the bandwidth of the amplifier was restricted by the use of a low pass filter and a parallel Tee filter. A 400 cycles per second power supply was used in order to eliminate the hum problem, which might have been quite difficult to solve had a 60 cycles per second power supply been used simultaneously with a 50 cycles per second modulation frequency. The overall gain of the voltage amplification stages is 84 db, plus or minus 1 db in the range 15 to 50 cycles per second. The noise level is approximately 48 db below a one millivolt input. Figure 21 is a block diagram of the amplifier, Figures 22a, 22b, and 22c are the schematic wiring diagram of the amplifier, and Figure 23 is the gain of the amplifier as a



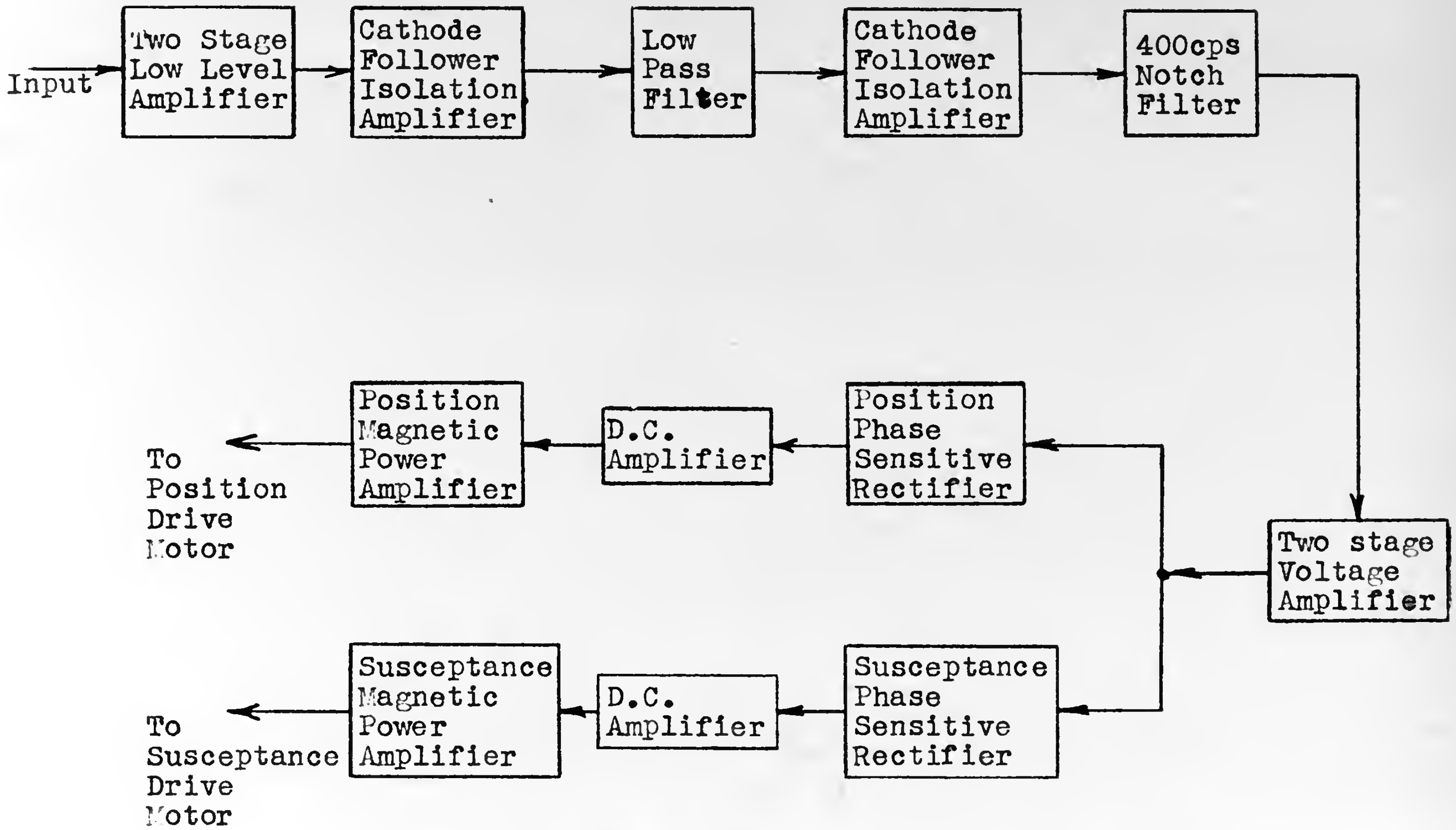


Figure 21

Servo Amplifier Block Diagram

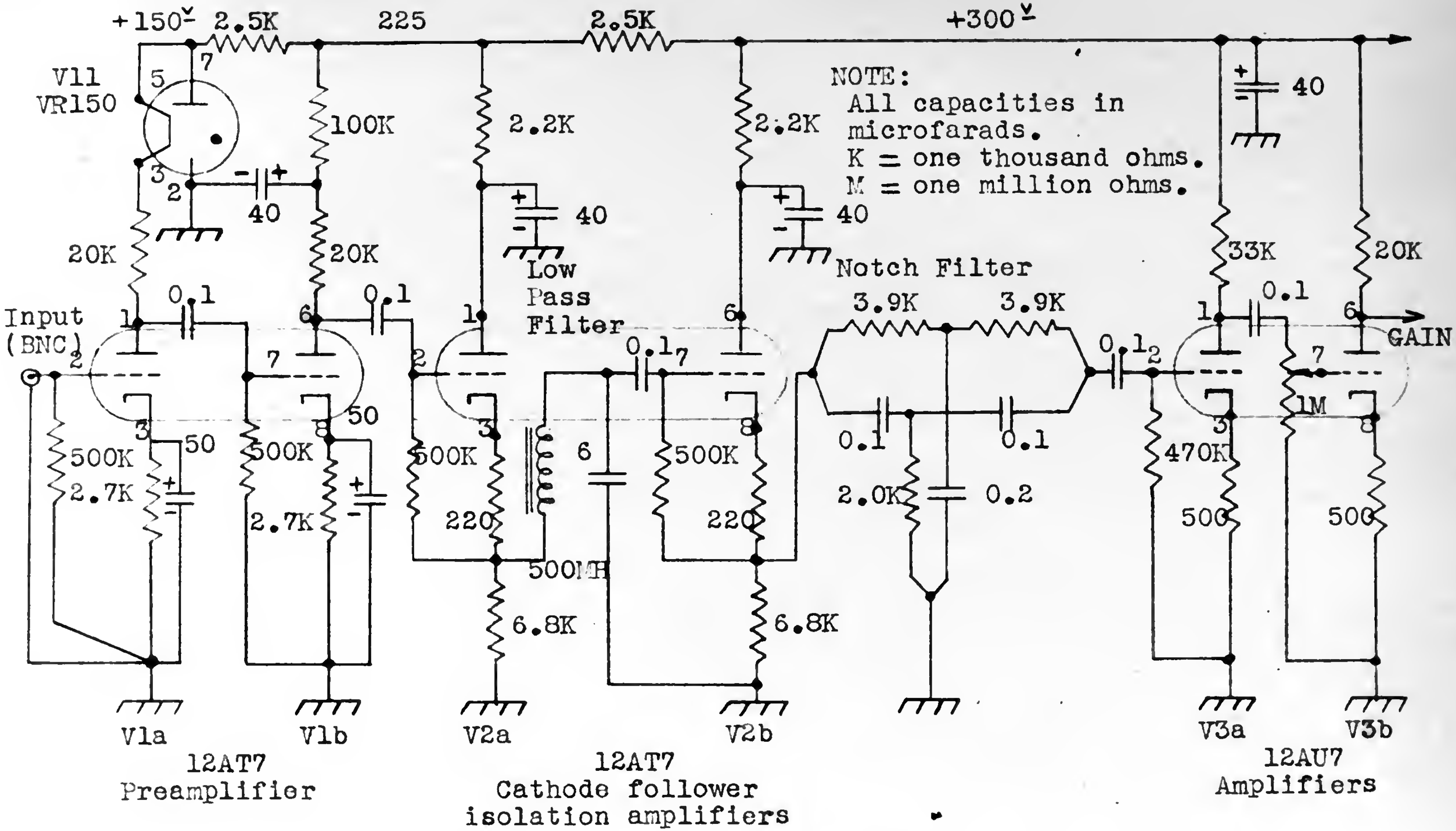


Figure 22a

Servo Amplifier Schematic - Low Level Amplifier and Filters

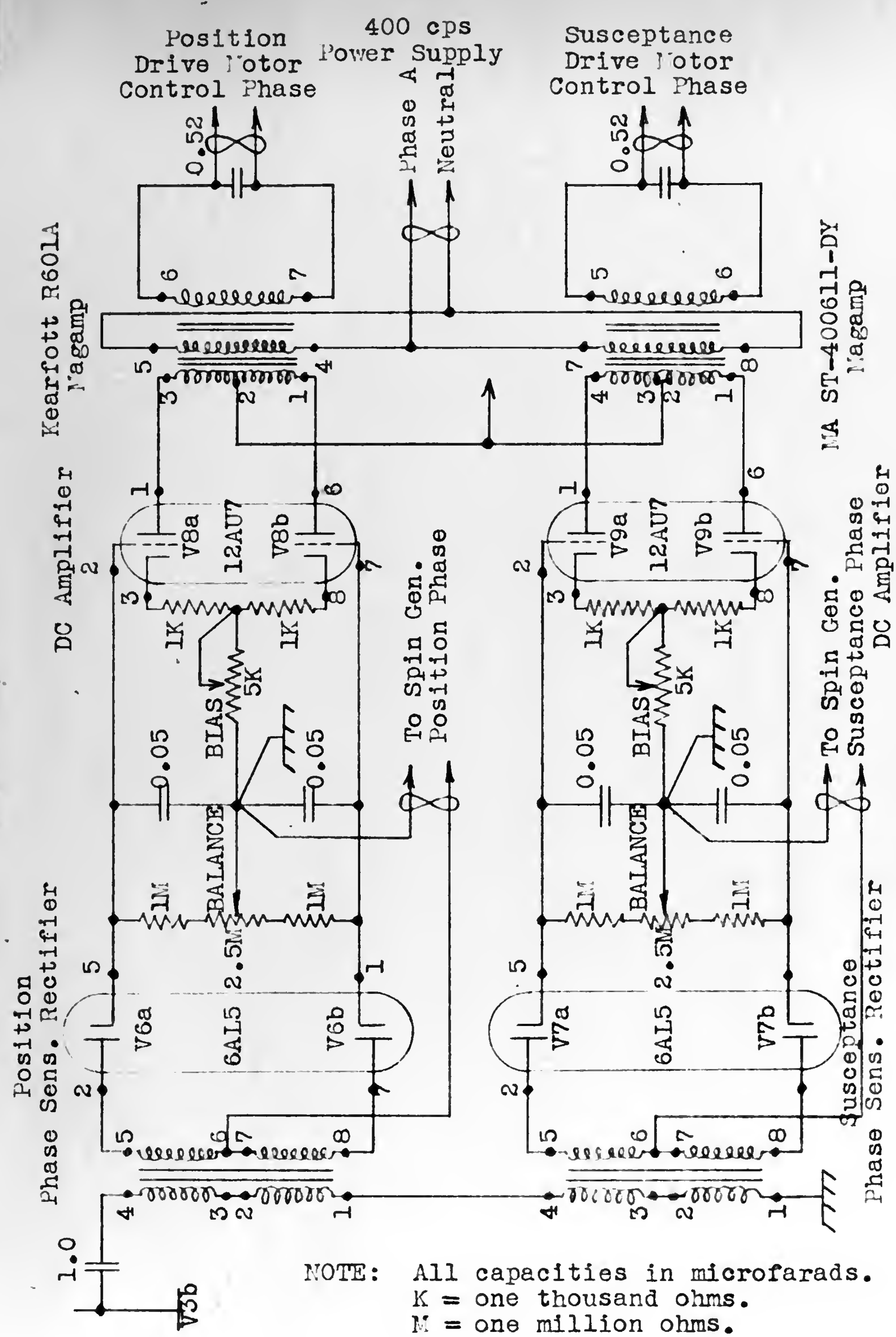


Figure 22b

Servo Amplifier Schematic - Detectors and Power Amplifiers

NOTE: All capacities in microfarads.
K = one thousand ohms.
M = one million ohms.

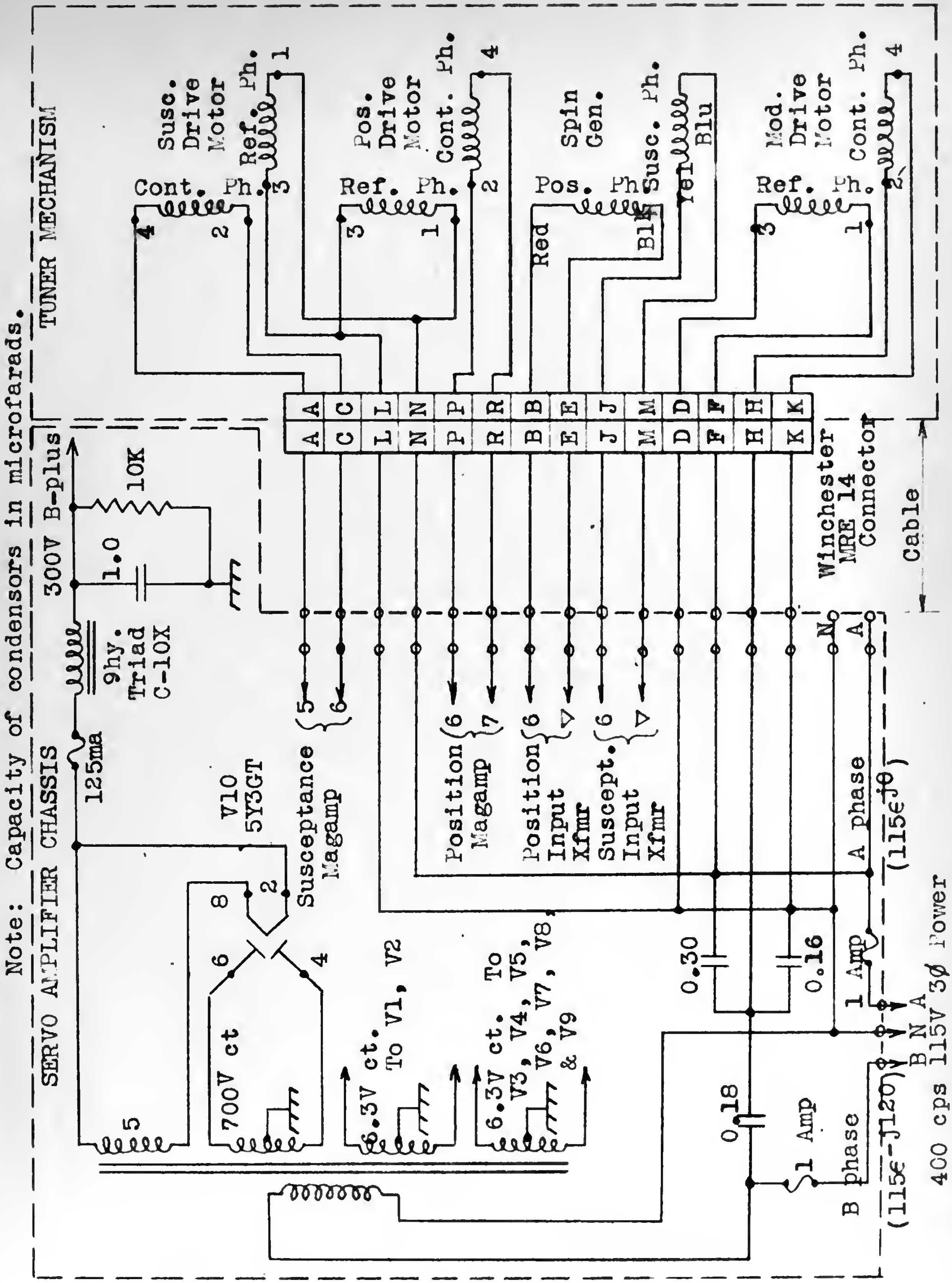


Figure 22c

Servo Amplifier Schematic - Power Supply and Cable

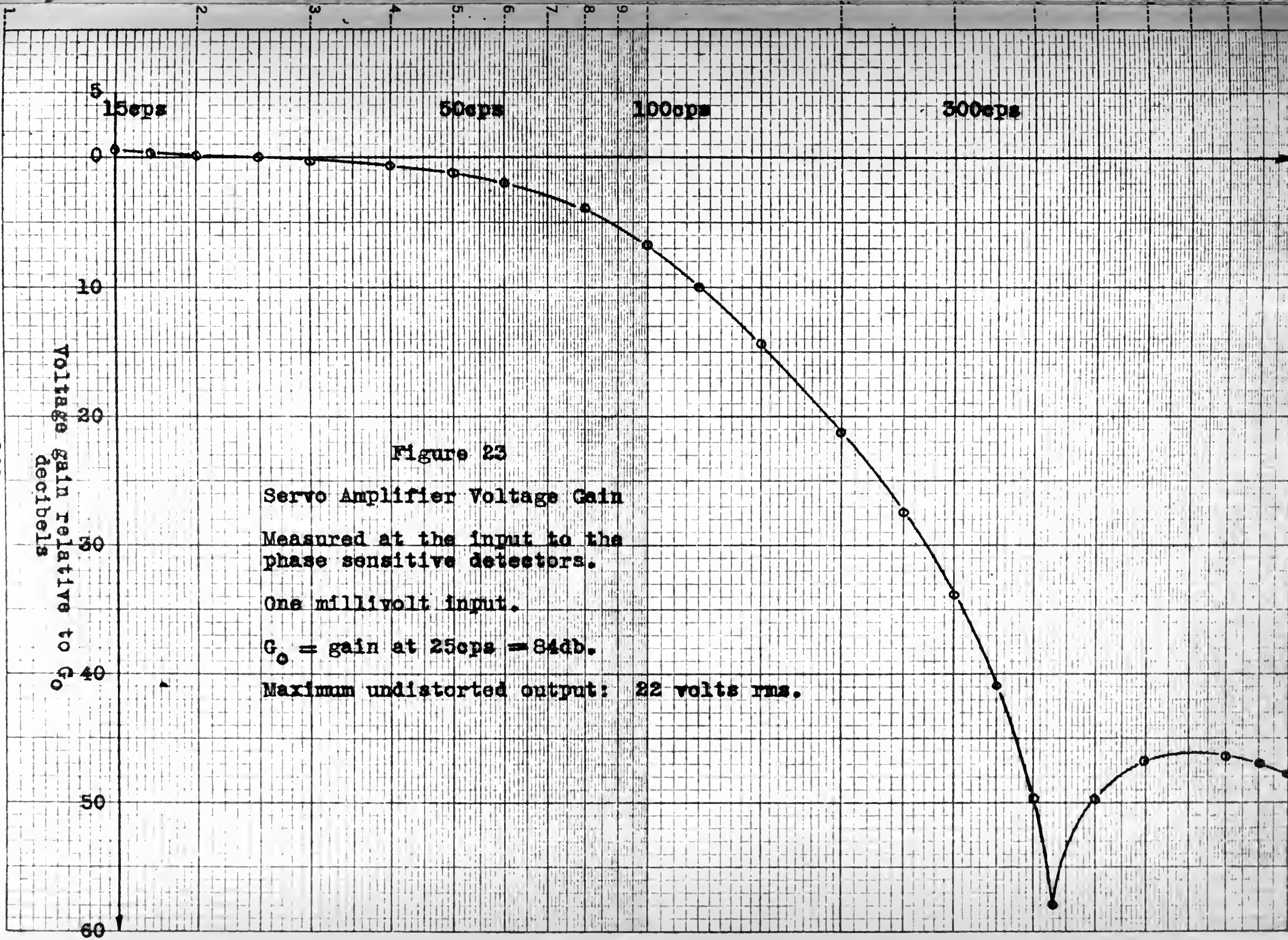


Figure 23

Servo Amplifier Voltage Gain

Measured at the input to the phase sensitive detectors.

One millivolt input.

G_o = gain at 25cps = 84db.

Maximum undistorted output: 22 volts rms.

function of frequency. The amplifier is pictured in Figure 16.

7. The positioning drive system.

The positioning drive system consists of a Mark 14, Mod 0 Kearfott servo motor and reduction gear train which drives the carriage along the waveguide by means of a rack and pinion drive on each side of the carriage. The positioning drive motor and gear box are pictured in Figure 16.

8. The susceptance drive system.

The susceptance drive system consists of a Mark 14, Mod 0 Kearfott servo motor and single reduction gear which rotates the susceptance probe shaft. This shaft is threaded at its upper end, and change of penetration is accomplished by means of a threaded collar in which the upper end of the shaft rides. The susceptance drive system is pictured in Figure 16.

9. Performance.

The Susceptance Probe Tuner automatically reduces values of load VSWR of 4.0 or less to less than 1.10 over the frequency band 8200 to 9400 megacycles per second. A matched load presented to the tuner results in the production of a VSWR of about 1.10, due to the mean value of susceptance of the susceptance modulation probe. The tuner has not been tested at frequencies lower than the above band. The upper limit of the frequency band of operation is due to frequency sensitivity in the susceptance probe mount.

The Susceptance Probe Tuner has not been completely

tested at this writing. Response times have not been determined, and so it is not known whether the tuner is capable of following cyclic variations of load such as are introduced by slowly rotating antennae of search radar. Some information relative to the effects of changes in the power level of the incident wave is also needed.

The Susceptance Probe Tuner has been operated using a pulsed klystron as power source. The duty cycle was not typical of radar applications, but pulse repetition rates from 400 to 1000 cycles per second resulted in normal operation of the tuner.

10. Conclusion.

In its present state, the Susceptance Probe Tuner shows a great deal of promise for use in the laboratory as an automatic matching transformer, and possibly for extension to use as an automatic admittance plotter; and in higher power applications as an automatic matching transformer. The present form of the tuner limits its power range due to the voltage breakdown characteristics of a metal probe inserted into the waveguide parallel to the E fields. However, a different form of susceptance device, possibly an E-plane or H-plane shunt arm with short circuit termination could eliminate this difficulty.

The operation of the Susceptance Probe Tuner is considered to be experimental verification of the analyses of this paper.

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